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The Constructs of Students' Logical Thinking in the Problems of Algebraic-Geometric Transformations

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Abstract: - Logical thinking is a cognitive ability that plays a vital role in solving problems. The art of logical thinking is recognizing, selecting, and applying the appropriate reasoning to solve a problem. This study aimed to identify and investigate students' logical thinking reasoning constructs when they face algebraic-geometric transformations. We used Kenexa tests to measure students' mathematical logical thinking. The statistical sample size was 380 students from high schools in Tehran, Iran. We applied exploratory factor analysis to identify and examine appropriate indicators and latent factors of logical thinking. The results showed that an adequate number of suggested indicators could identify the reasoning dimensions of students' mathematical logical thinking, and it can be used as a logical reasoning test in algebraic-geometric transformations. Moreover, the different performance of students in using reasoning strategies indicates that the content of textbooks and the teaching approach in the field of algebraic-geometric transformations should move in a direction that focuses on reasoning skills.

Keywords: Algebraic-Geometric Transformations. Logical Thinking. Logical Reasoning.

I. INTRODUCTION

Logical thinking refers to the idea that the study of mathematics can prove certain things are correct and that a specific rule exists that can organize mathematics-related concepts (Macdonald, 1986). Researchers have recommended and suggested paying more attention to developing mathematical thinking skills, especially logical thinking, by applying techniques that integrate formative evaluation into the teaching process (Jawad and Majeed, 2021).

Researchers such as Hill (1960), Lawson (1978), Tobin and Capi (1981), Roadrangka et al. (1982) and Leongson and Limjap (2003) provided tests to evaluate the ability of logical thinking. They explained the reasoning components in logical thinking, considering the stages of cognitive development. The proposed tests do not result from learning or relate to specific research branches. The Naglieri test (Lee et al., 2021) and Raven's Progressive Matrices (Raven, 1989) are non-verbal tests used in educational settings to diagnose gifted students or students with learning disabilities. Although these tests are widely used to measure general cognitive ability, they do not assess general cognitive ability as the primary variable of interest but as a background variable (Langener et al., 2022). We intend to analyse students' logical thinking as a primary variable and identify its latent structures, specifically in the face of algebraic-geometric transformations.

Considering the role of logical-mathematical intelligence in students' performance, it seems necessary to design and validate logical thinking tests in different areas of mathematics education. On the other hand, Bronkhorst et al. (2019) identified that students' reasoning strategies in different assignments can increase the teacher's awareness of the various strategies and guide the class discourse during logical reasoning courses. Kaput (2008) presented the structures and systems abstracted from relations and operators as one of the fundamental aspects of school algebra. Clements and Battista (1992) explained that the study of formulated transformations is essential in school geometry.

Carraher and Schliemann (2014) mentioned that algebra reasoning relates to psychological processes during solving a problem by applying algebraic symbols, which are subjective. Bodanskii (1991) suggested that algebraic methods are a natural and more effective way to solve problems than arithmetic methods. Although 'geometric algebra' was discovered by William Kingdon Clifford (1878), it remains far from the main center of mathematics

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(Pozo and Sobczyk, 2002). One of the 'geometric algebra' in school mathematics is algebraic-geometric transformations. Logic is the science and art of accurately finding the truth and teaching reasoning (Luna- Guevara et al., 2021). For a long time, logical thinking and reasoning have been considered a goal of mathematics education (Suydam, 1985).

Intuitive claims about abstract concepts such as logical thinking and its various constructs in facing problems, especially assignments related to different disciplines in mathematics education, require empirical evidence. Therefore, in this research, we identify appropriate indicators and latent factors of logical thinking by implementing a logical thinking test. Then, we introduce the constructs for a structural model of algebraic-geometric transformations to define five kinds of reasoning.

Literature Review and Theoretical Framework

Logical Thinking

There are various ideas about logical thinking. This thinking includes the mental processes that a person uses when dealing with certain puzzles. Some researchers have introduced logical thinking abilities as a necessity to overcome daily obstacles in life (Linn et al., 1981). Logical thinking moves from known to unknown, following the rules and standards of specific goals with logical grammar (Shatnawi, 1982). According to Lawson (1992), one of the requirements of conceptual learning is the application of logical thinking. When an individual's logical ability develops, conceptual knowledge will also develop.

Logical thinking is one of the intellectual skills for problem -solving. In this context, this type of thinking relates to skills such as effective use of numbers, identification of analytical nexus between concepts, classification, generalization, creation and analysis of hypotheses, and calculation through mathematical formulas (Bektasli, 2006). Korkmaz (2012) believes mathematical thinking and logical intelligence are associated with developing algorithmic skills. He examined in a case study the effect of problem-solving and cognitive and logical thinking skills on the score of students in a C programming course (Baidowi et al., 2013).

Gardner (2013) believed that logical-mathematical intelligence is finding associations or discussing patterns and sequences. A logical thinking method is required to understand patterns and sequences. Students' thinking methods may consequence from their suitable logical intelligence. Incikabi et al. (2013) found a weak and negative correlation between mathematics teachers' critical thinking positioning and their logical skills. Logical thinking requires constant thinking to conclude. The sequential thinking process exists at the heart of logical thinking. It gathers all the facts, ideas, and results and organizing in one chain (Aksu and Koruklu, 2015). The logical thinking ability may be obtained from learning activities while acquired through Euclidean geometrics and logical reasoning in mathematics (In'am, 2016). Ozdemir and Ovez (2017) showed that teachers' logical thinking skills and approach make a significant difference among the levels of geometric proof writing.

Building a bridge between the world of algebra and logic is the methodology of solving logical problems by translating them into algebra (algebraization process); solving algebraic problems and translating the result is one of the broad topics (Andréka et al., 2001). Logical thinking ability is achieved as competency in mathematics. Mathematics learning activities can improve logical thinking abilities (Ab et al., 2019). Logical thinking is the ability to do logical operations and informed reasoning (Samadovna et al., 2020).

In recent studies, based on students' cognitive development stages, mathematical reasoning has been analyzed and tested with a particular learning style (Gunawan et al., 2019). Also, the impact of teaching mathematics on logical thinking leads to better reasoning in a cross-sectional assessment from high school to university (Cresswell and Speelman, 2020). Yunus (2021) believed that logical thinking is one of the characteristics of mathematical thinking and should consider for educating mathematics students from elementary school. In describing logical thinking as a mathematical mental process, he mentioned the features of reasoning, analysis, communication, comparison, grouping, distinguishing between similar things, choosing rules, and appropriate actions. He applied the Posse strategy to develop students' logical thinking in the transition from the concrete cognitive development stage to the abstract stage. Also, Mohammed and Hassan (2021) investigated the effect of the Posse strategy on characteristics and skills of logical thinking, such as types of reasoning, induction, comparison, prediction, and innovation. Logical thinking and reasoning in mathematics and science are essential elements for forming social communication, students' identity, and developing higher-order thinking (Shuib et al., 2021).

Algebraic-Geometric Transformations

Pozo and Sobczyk (2002) mentioned that geometric algebra was discovered in 1878 by Clifford. In addition to showing new ways to reason in geometry, the term 'geometric algebra' provides mathematicians with the necessary tools to interpret geometry and its relationships (Suter, 2003). However, Perwass and Hildenbrand (2004) clarified that this term refers to the geometric interpretation of algebraic objects and is considered a field of vector algebra. According to Plato, mathematical objects are abstract, incomprehensible, and even ideal entities of the senses (Gerstberger, 2009).

Transformation is one of the topics in geometric algebra. Geometric objects, such as points, lines, and planes, are represented by algebraic components, which is geometric algebra. Then, geometric operations on objects, such as rotation, translation, and reflection, are represented by algebraic operations, which are geometric algebra (MacDonald, 2017).

Hollebrands (2003) considered algebraic-geometric transformations in mathematics as a context for viewing an interconnected discipline, an opportunity to learn new ways of thinking about mathematical concepts and using different representations in high-level reasoning activities. Algebraic-geometric transformations have effectively discovered patterns, created generalities in visualization, and developed spatial, logical, and reasoning thinking (Yanik, 2014). Transformations can make students interested in searching through mathematics abstract concepts, such as congruence, symmetry, similarity, and parallelism, enriching students' algebraic-geometric experiences, thoughts, and imagination. Therefore, transformations can improve students' logical and spatial abilities (Guven, 2012). Through a comparative study, Thaqi and Gimenez (2016) analyzed the topic of transformations in the content of elementary mathematics textbooks in two countries, Spain and Kosovo. The results showed the difference between the examples presented in the textbooks, the students' performance, and the teachers' approach to the teaching of transformations. Also, they emphasized different aspects of transformations, including types, hierarchy, mathematical communication, reasoning, and historical and cultural elements in curriculum and complete implementation by teachers. Mukamba and Makamure (2020) examined GeoGebra as a suitable dynamic tool for learning and teaching algebraic-geometric transformations. The results showed the positive effect of using the dynamic tool in learning and teaching. According to Fife et al. (2019), looking at transformations with a geometric algebra approach as functions of the plane will reveal a qualitative change in a concept's complexity level and mental habits.

Tests to Measure Logical Thinking and Reasoning

Inhelder and Piaget (1958) conducted some experiments and showed that logical thinking reaches its main point during the concrete operational stage (7-11 years) and formal operational stage (11-18 years). Following Piaget's theory of cognitive development, logical thinking refers to the ability observed in the concrete and abstract operations stage. Piaget believes understanding formal rules in decision-making is the base for hypothetic-deductive thought, which appears during adolescence. Researchers have provided tests to assess logical thinking ability based on the underlying features of thinking in Piaget's formal stage, which is as follows:

Hill's test (1960) was designed to measure logical thinking based on the following components: sentential, hierarchical analogy, and quantitative logic (O'Brien & Shapiro, 1968). Test of Logical Thinking (TOLT) comprised one component as "controlling variables" and four reasoning components: "proportional reasoning", "probabilistic reasoning", "correlational reasoning" and "combinatorial reasoning" that Tobin and Capie (1981) covered ten 2-option cases with reasoning. Similarly, the classroom test of formal reasoning (Lawson, 1978) was designed considering three components: "proportional reasoning", "probabilistic reasoning", and "correlational reasoning" (Kamaruddin et al., 2004).

Roadrangka et al. (1982) designed the Group Assessment of Logical Thinking (GALT), introducing six components (combinatorial reasoning, correlational reasoning, proportional reasoning, probabilistic reasoning, eternity (conservation) reasoning, and controlling variables), which covered six models of Piaget's thinking. TOLT, GALT, and Lawson tests are not learning result tests and do not deal with specific research contexts. Leongson and Limjap (2003) assessed classification, sequence, logical multiplication, compensation, proportionality, probability, and correlation in a reasoning model they developed to evaluate students' achievement in higher education mathematics. They introduced the Test of Logical Operations (TLO) in mathematics, which is confined to the majority of mathematical courses in high school, including geometrics, mathematics, statistics, and algebra. In the dimensions of algebraic thinking, three forms of reasoning are considered: inductive, deductive, and abductive (Chimoni et al., 2018). In addition to the reasoning components mentioned in the above tests, in a more general view of mathematical thinking, arguments such as visual and spatial reasoning are also essential and closely related to geometry (Clements and Battista, 1992). In Gardner's (2013) classification, spatial reasoning is a manifestation of spatial intelligence and thinking. In the current study, the assessment of students' logical thinking in the face of algebraic-geometric transformations in two-dimensional space is considered, so the test questions have been selected with a focus on reasoning components, as follows:

Proportional Reasoning

A good understanding of different functional relations in mathematics requires proportional reasoning. Proportional thinking refers to relating a component to another component or a whole regarding the magnitude or degree (Leongson and Limjap, 2003).

Combinatorial Reasoning

Combinations are described as the skill of counting all methods in a certain number of entities that can be integrated so that no result is missed (Batanero et al., 1997).

Correlational Reasoning

Leongson and Limjap (2003) define correlational thinking as creating a correlation or causal relation. Moreover, they describe correlational reasoning as a method that "Can reason with relationships of variables or symbols" (p. 8). Correlational reasoning is the capacity to connect two distinctive associations between various statuses and comprehend that if something transformations in one connection, it will also transform in another (Dugan, 2006).

Inductive Reasoning

Inductive reasoning is a method to discover phenomena' characteristics and find regularities logically (Polya, 1973). Inductive reasoning in mathematics education is a reasoning process that starts with specific cases and creates a generalization from these cases (Cañadas and Castro, 2007).

Mathematical inductive reasoning involves the process of drawing details to general conclusions and estimating answers or solutions (Kurniawati et al., 2021). Three features are considered for inductive reasoning: (1) it draws from particular cases to draw general conclusions, (2) it uses what is known to draw conclusions from the unknown, and (3) the outcome is often probable and not imperative and certain (Sosa-Moguel and Aparicio-Landa, 2021).

Analogical Reasoning

The mathematical analogy is a conclusion based on similarities between observed procedures (Sumarmo, 2013). On the other hand, Analogical reasoning searches to find similarities or proportions in some characteristics between two situations or phrases used for comparisons. Marcus (1987, p. 91) said: "Analogical reasoning is one of the most powerful mathematical thinking tools." Any argument based on analogy can be considered analogical reasoning. One of the primary uses of analogical reasoning is in transferring abstract mathematical concepts (Gentner et al., 2001).

One mathematical concept mentioned in the analogy is the analogy of a cube volume with a square volume (Nappo et al., 2023). A sample of the effectiveness of conceptual analogy in the design of building materials is shown in Fig 1.

This study focuses on Piaget's theory of cognitive development and the different components of logical thinking proposed in Piaget's theory. It is attempted to identify the latent structures of students' logical thinking, particularly when faced with the problems of algebraic-geometric transformations, by presenting an exploratory model based on factor analysis to expand the existing literature in this field.

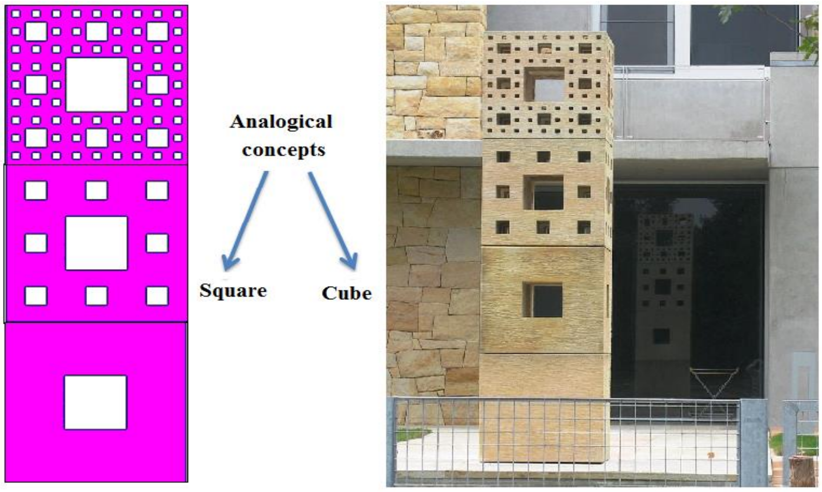

Fig. 1 *A sample of the effectiveness of conceptual analogy in the design of building materials*

Questions of the Research

This research sought to validate the indicators of the logical thinking test proposed based on Piaget's theory of cognitive development. It also investigates and analyzes the latent reasoning factors of logical thinking and the indicators related to each factor. Determining the weight of reasoning components that students use with algebraic-geometric transformations can increase the teacher's awareness of the variety and students' reasoning indicators in the context of transformations. Therefore, explaining a visual model of logical thinking structures can be used as a practical guide for teaching algebraic-geometric transformations.

The following questions are answered in this research.

1- Which indicators of the test are suitable for measuring the mathematical logical thinking of students in the face of algebraic-geometric transformations?

2- How is the structural model of students' mathematical logical thinking when solving algebraic-geometric transformation problems?

II.METHODOLOGY

This study used a quantitative research plan of descriptive-analytical type. The correlational study was conducted to identify any possible correlation between two or more variables and determine their importance and direction (Creswell, 2008). Watkins (2018) describes exploratory factor analysis (EFA) as "a multivariate statistical method that attempts to identify the smallest number of hypothetical constructs (also known as factors, dimensions, latent variables, synthetic variables, or internal attributes)" (p. 219). Therefore, this research used EFA to examine the correlation between variables.

The first-order factor analysis was applied to investigate the correlation between the test's questions (observer variables) and reasoning factors (latent variables). Then the second-order factor analysis was used to find the correlation between reasoning factors and mathematical logical thinking. A latent variable is a variable that underlies observations and is not directly measured. When a variable is hypothesized as latent, it means that the desired inference cannot make with certainty. Variables are not inherently latent or manifest. They can only be considered latent or evident according to the available data or, in other words, according to the observer and his measurement equipment (Borsboom, 2008).

Participants

The statistical sample comprised 380 participants (230 female and 150 male students) selected using purposive criterion sampling. Criterion sampling examines cases with several pre-specified criteria (Yıldırım and Simsek, 2005). This study had two criteria: participants were selected from eleventh or twelfth-grade students studying in public schools during the academic year of 2021-2022. The selected students were taught course contents of geometric transformations and algebraic functions.

It should be noted that the total number of participants was (n=423) students of the eleventh or twelfth grades of secondary school, of which 43 cases were discarded as outliers (When we checked the students' answers and the time of submission, we realized that some chose the answer options randomly. This data is invalid and will overshadow the results; therefore, we excluded them from the analysis as outliers.), and 380 samples were considered for statistical analysis (Table 1), which shows the demographics of the research participants.

Table 1. *Frequency distribution of selected students*

Variable	Category	Number	Percentage(%) *
Gender	Female	230	60.5 %
	Male	150	39.5 %
Grade	eleventh	256	67.4 %
	twelfth	124	32.6 %
Age (years)	16-17	261	68.7 %
	18-19	119	31.3 %

Note. *= Frequency percentage is rounded with a maximum error: 0.03.

Instrument of Study

The instrument of the present study is a practice test containing 20 questions (indicators) that are extracted from the logical thinking tests of Kenexa. Two math professors and four teachers validated the questions (indicators), subject matter experts in algebraic-geometric transformations with at least 15 years of teaching experience. The content structures of the indicators were determined through the analysis of algebraic operators and geometric transformations used for each question. Content structures include functional operators 1) translation, replacement, 2) reflection, 3) dilation and size change, and 4) rotation and their combination. Each content structure relates to geometric transformations such as translation, reflection, rotation, and dilation, and algebraic operators such as shape change mapping, colour, size, and their combination that are considered in two items (indicators). Since measured each construct twice, the 20 indicators were acceptable for measuring the logical thinking of students for the issue of algebraic-geometric transformation, which is operationalized as their complete score on the test and determined the cognitive domain awareness to ensure that all types of reasoning related to logical thinking (the latent structures of the concept of logical thinking) are covered by default in the test subjects.

According to Bloom's revised 6-level taxonomy system, it was determined the cognitive domains involved and the behavioral goals predicted for problem-solving by the student.

When selecting the questions, the percentage distribution of cognitive domains was considered according to the proposed model (LO/IO/HO) by Sivaraman and Krishna (2015). Students need to identify and recognize algebraic-geometric transformations (remembering), predict the answer by illustrating and classifying symbols and operators (understanding), implement and execute different algebraic-geometric transformations based on observed processes (applying), and determine how the given operators and signs are associated with each other (analyzing), monitor the logic to puzzle out the problem (evaluating), and ultimately hypothesize the procedure that attended to the answer (creating). Therefore, the test questions of logical thinking consider cognitive domains that cover all six levels of Bloom's revised taxonomy. The content structures of algebraic-geometric transformations, cognitive domains involved, and behavioral goals related to logical reasoning to solve questions of the test are summarized in Table 2.

The consensus between professionals for content coverage and items' relevancy was computed using item-level and scale-level content validity indices. Since there were six experts in the validation panel, used the S-CVI/Ave method for the content validity index at the scale level (Polit and Beck, 2006). The value of S-CVI/Ave was calculated as 0.916, and the value of I-CVI was equal to 0.915. With the S-CVI/Ave transcending 0.90 and the I-CVI transcending 0.78, the content coverage pertinent to the logical thinking test was reasonable, and all the items were relevant to the issue of algebraic-geometric transformations covered in the Grade 11 Mathematics textbook (Lynn, 1986).

Table 2. *The content structures of algebraic-geometric transformations, cognitive domains involved, and behavioral goals related to logical reasoning in the test*

Questions (Indicators)	Content structure related to algebraic-geometric transformations	Cognitive domain	Behavioral goals

Q_1	Translation-Reflection of the shapes in the X axis	evaluating	The student can combine variables to create all possible combinations and draw conclusions.
Q_2	Mapping of shape color change	understanding	The student can reach a general conclusion based on limited evidence.
Q_3	Simultaneous mapping of changing color and position of symbols	creating	The student can reason with cause-and-effect relationships between variables or symbols.
Q_4	Rotation of symbols 90 clockwise or counterclockwise	evaluating	The student can combine variables to create all possible combinations and draw conclusions.
Q_5	Mapping of changing the shape, color, and form of symbols	analyzing	The student can reach a general conclusion based on limited evidence.
Q_6	Translation	understanding	The student can reach a general conclusion based on limited evidence.
Q_7	Mapping of changing the shape, color, and form of symbols	applying	The student can combine variables to create all possible combinations and draw conclusions.
Q_8	Reflection of the shapes in the Y axis	remembering	The student can reason with cause-and-effect relationships between variables or symbols.
Q_9	Mapping of changing shape and color of symbols	applying	The student can reason with cause-and-effect relationships between variables or symbols.
Q_{10}	Rotation of symbols 90 clockwise or counterclockwise	evaluating	The student draws conclusions based on similar observed processes.
Q_{11}	Color change mapping according to form and place of shape	applying	The student can combine variables to create all possible combinations and draw conclusions.
Q_{12}	Dilation	analyzing	The student can understand the different functional relations in mathematics and the relative value of decreasing and increasing ratios.
Q_{13}	Translation	remembering	The student can reach a general conclusion based on limited evidence.
Q_{14}	Dilation	understanding	The student can understand the different functional relations in mathematics and the relative value of decreasing and increasing ratios.
Q_{15}	Reflection of the shapes in the Y axis	remembering	The student draws conclusions based on similar observed processes.
Q_{16}	Mapping of changing shape and color of symbols	analyzing	The student draws conclusions based on similar observed processes.
Q_{17}	Shape change mapping according to color and size and form of symbols	analyzing	The student can understand the different functional relations in mathematics and the relative value of decreasing and increasing ratios.
Q_{18}	Shape change mapping according to color and size and form of symbols	creating	The student can understand the different functional relations in mathematics and the relative value of decreasing and increasing ratios.
Q_{19}	Translation and mapping of changing position of symbols	applying	The student draws conclusions based on similar observed processes.
Q_{20}	Color change mapping according to form and place of shape	creating	The student can reason with cause-and-effect relationships between variables or symbols.

To determine the reliability of the test, we conducted a pilot study with 35 students in grade 11. Then, we calculated the reliability of the pilot study by the value of Cronbach's alpha. The value of 0.81 on the logical thinking test demonstrated a good internal correlation. With internal consistency surpassing 0.7, the logical thinking test was reliable for the actual study (Taherdoost, 2016).

An indicator (question) and a sample of students' answers with the behavioural objective of proportional reasoning are given below (Fig 2).

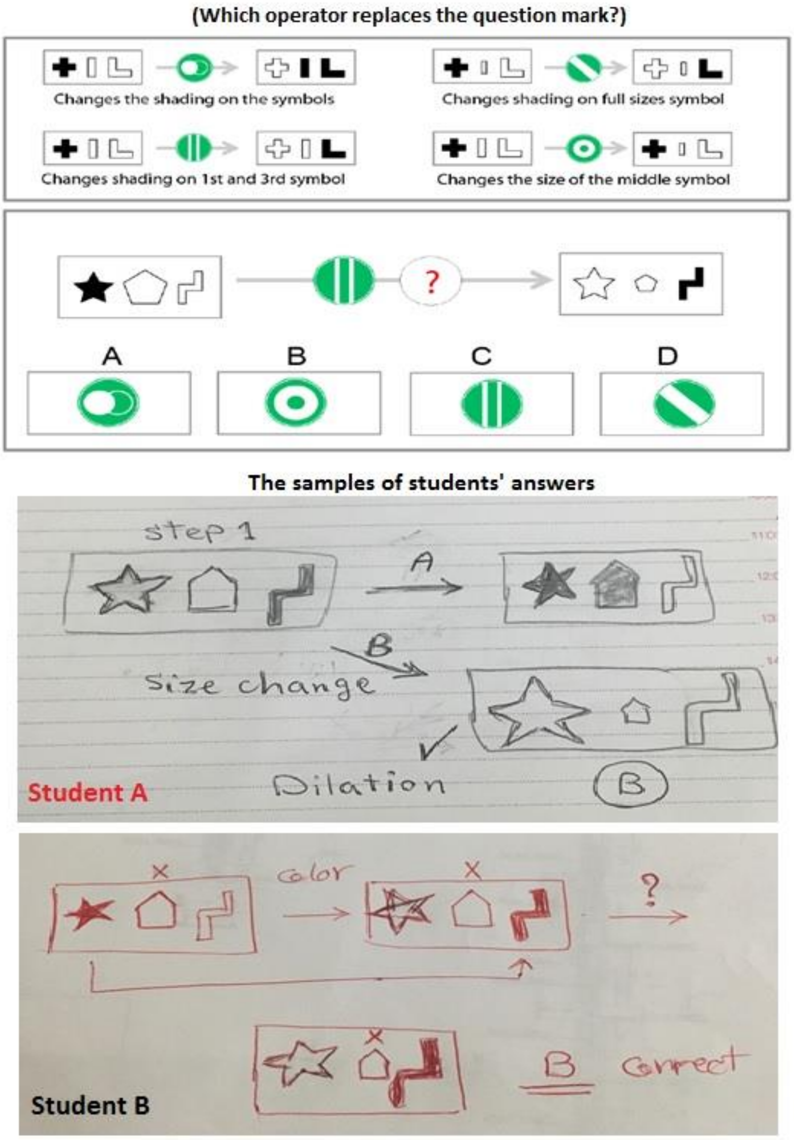

Fig. 2. *A sample of student's answers to the mentioned question from the category of proportional reasoning*

Dilation is a type of nonrigid transformation in which the image and preimage of the figure are similar. Dilation in Geometry is a geometric operation that introduces a scale factor determining the degree of enlargement or reduction applied to a figure (Larson et al., 2004). As it follows from the definition, Dilation is a proportional change in a shape's size. The length of each side in the shape is multiplied by the same number, but the shape and its angle measures remain the same.

Research procedure

Since the first condition for the exploratory factor analysis statistical process, the number of statistical samples is acceptable. Therefore, it was coordinated with 15 teachers and principals of the schools who taught there to compile the number of statistical samples necessary to implement the test. After obtaining permission from the participating schools' authority and the participating students' parents, the teachers administered the logical thinking test during classroom hours. Before the start of the test, the necessary instructions to answer the questions were explained by the teacher to the participants so that they could accurately focus on the answer during the test. The duration of the test was 30 minutes, which was considered according to the number and type of questions and the consensus of the experts. In each school, the test was administered to the students at the time of school hours that the teacher determined, so the process of the test execution and data collection lasted for almost four weeks.

The test was administered in a virtual mode in the conditions of the Covid-19 epidemic, and the response time of each student to the test was recorded. Students' scores were determined based on the number of correct answers to the test questions from score: 0 to score: 20. By investigating the duration of answering the test and the scores obtained, some data (43 data) were identified as outliers and excluded from the analysis. Exploratory factor analysis was used to identify latent factors and explain the degree of correlation between indicators and latent factors related to the abstract concept of logical thinking. For exploratory factor analysis, the score of each question was coded as the correct answer (2), wrong explanation (1), and no response (0). The Statistical Package for the Social Sciences (SPSS) was used to analyze the data.

III.RESULTS AND DISCUSSION

EFA was done on the studied sample data (n=380) to identify and interpret latent factors in students' logical thinking. We measured Kaiser-Meyer-Olkin (KMO) value before EFA to determine whether the sample size was reliable.

KMO of sampling adequacy equaled 0.772, surpassing 0.7, and Bartlett's test criteria were checked (Table 3), confirming the proper relation between data structures. The results indicated EFA feasibility.

Table 3. *Sampling Adequacy*

KMO-SA		0.772
TS	χ^2	3658.372
	df	91
	sig	0.000

Note. KMO-SA= Kaiser-Meyer-Olkin of Sampling Adequacy

Note. TS= Bartlett's Test of Sphericity

Based on EFA (See Table 1 in appendices.), of 20 initial indicators, 14 were extracted, and their coefficients of determination equaled 0.5 or greater.

The total explained variance equaled 68.33%, and we identified five factors with eigenvalues greater than 1. The first, second, third, fourth, and fifth factors explained around 21%, 15%, 13%, 11%, and 8% of the variance, respectively. These five factors could explain around 68% of the variables' variance (Table 4).

Table 4. *Total Variance Explained*

components	ES -SL			RS-SL		
	Total	% of Variance	%Cumulative	Total	% of Variance	%Cumulative
1	3.815	27.253	27.253	2.996	21.401	21.401
2	2.321	16.579	43.832	2.112	15.085	36.486
3	1.210	8.642	52.475	1.883	13.452	49.938
4	1.147	8.193	60.668	1.479	10.566	60.504
5	1.073	7.667	68.334	1.096	7.830	68.334

Note: ES-SL=Extraction Sums of Squared Loadings

Note: RS-SL= Rotation Sums of Squared Loadings

In the present analysis, we identified five components and one total factor for students' logical thinking. The mentioned components and factors were coded based on indicators and research literature.

Due to Table 5, factor loads surpassed 0.5. Hence, the extracted indicators are loaded into relevant factors. On the other hand, discriminant validity was confirmed since the correlation between each indicator and other factors was less than 0.5.

Table 5. *Ordered factor rotated matrix*

	factors				
	R_1	R_2	R_3	R_4	R_5
Q_4	0.985	0.039	0.105	0.095	0.024
Q_7	0.982	0.043	0.099	0.089	0.021
Q_{11}	0.982	0.037	0.113	0.094	0.019
Q_{14}	-0.005	0.716	0.013	0.121	0.189
Q_{15}	0.097	0.676	0.194	-0.143	-0.033
Q_{17}	0.046	0.696	0.258	0.133	0.113
Q_{10}	0.035	0.054	0.692	0.146	0.327
Q_{12}	0.167	0.202	0.745	0.140	-0.064
Q_{18}	0.094	0.227	0.779	-0.008	-0.164
Q_2	0.166	-0.101	0.300	0.552	0.279
Q_5	0.135	0.119	0.100	0.682	-0.273
Q_6	0.024	0.228	0.001	0.721	0.082
Q_{16}	-0.018	-0.134	0.058	0.226	0.685
Q_{20}	0.038	0.112	-.002	-0.016	0.854

The students' answers to the loaded indicators in the latent factors of logical thinking were shown (See Table 2 in the appendices). Students' response rate in each of the factors shows different levels of performance when faced with algebraic-geometric transformations and using various structures of logical thinking (Table 6).

Table 6. *The* response *rate of students in each of the factors*

Logical Thinking Constructs	Frequency percentage of answers		
	True	False	No answer
Proportional Reasoning	86.03	13.00	0.97
Combinatorial Reasoning	76.40	22.99	0.61
Correlational Reasoning	59.35	39.05	1.6
Inductive Reasoning	76.58	22.72	0.70
Analogical Reasoning	57.90	40.26	1.84

First research question

Which indicators of the test are suitable for measuring the mathematical logical thinking of students in the face of algebraic-geometric transformations?

The findings of the exploratory factor analysis based on the principal component analysis method showed that there is a proper relationship between the data structure (significance level in Bartlett's test is less than 0.05). On the other hand, the coefficient of explanation of the variance of the indicators (questions) showed that some indicators do not have much relationship with any of the factors. Then, we should remove the indicator and perform the analysis again to achieve an underlying structure based on the relationship between the data (Hadi et al., 2016). Therefore, according to the table of commonalities, we confined one of the acceptable modes by repeating the analysis of removing inappropriate indicators (questions) and reduced indicators (questions) to 14 items.

Following the analysis process, with the help of cumulative variance explanation, the number of factors and the amount of variance explained for each factor were determined. Thus, the first five factors with eigenvalues greater than one and representing more than 60% of the variance of the variables (Table 4) will remain in the model. The component rotation matrix was obtained using the Varimax rotation method to interpret and identify the factors related to each index. Each row of the component rotation matrix corresponds to an indicator (question) and shows the correlation coefficient of the indicator (question) with factors. In this way, each indicator is loaded in the factor with the highest correlation.

The construct validity of the model's latent variables was used to evaluate the validity of the measurement model. The convergent validity was done with the factor load criterion. The discriminant (divergent) validity was determined by statistical analysis based on the correlation between variables and relevant factors and the correlation between each factor and other factors. Divergent validity establishes when each indicator (question) has a correlation value of more than 0.5 with the factor in which it is loaded and a correlation value of less than 0.5 with other factors. If an indicator (question) has a high correlation with two factors, that indicator (question) should be removed from the model (Yu and Richardson 2015). The result showed that divergent validity is established, and no need to remove any of the 14 indicators (questions).

Second Research question

How is the structural model of students' mathematical logical thinking when solving algebraic-geometric transformation problems?

The mentioned components are coded as proportional reasoning, combinatorial reasoning, correlational reasoning, inductive reasoning, and analogical reasoning regarding the nature of indicators and research literature. Second-order exploratory factor analysis was used to achieve a broader concept (logical thinking) of these five components. The average indicators (questions) related to each of the five components were calculated and replaced by that component and considered new inputs for factor analysis. As can be seen from the first and second order EFA results, we can model the structural pattern of students' logical thinking (Fig 3).

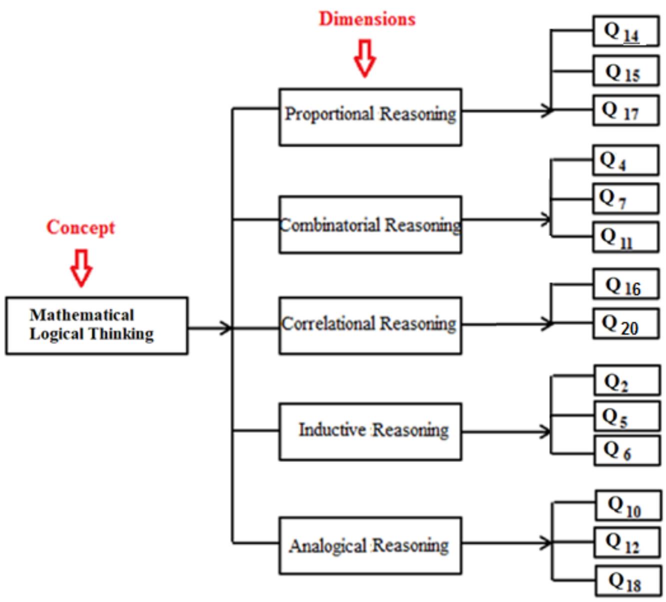

Fig. 3. *The model is designed for students' logical thinking Based on the (first and second order) EFA*

IV.CONCLUSIONS

The research results showed that reducing the indicators (questions) of the designed logical thinking test is possible. The indicators are reduced to 14 items. Factor analysis and the Varimax analysis method showed that the extracted questions can be used to evaluate students' logical thinking structures in algebraic-geometric transformations.

Data from the present study and their exploratory factor analysis show that students use different components of logical thinking, such as proportional reasoning, combinatorial reasoning, correlational reasoning, inductive reasoning, and analogical reasoning when dealing with algebraic-geometric transformations. This finding matched the results obtained by Roadrangka et al. (1982), Leongson and Limjap (2003), and Sumarmo et al. (2012). On the other hand, the model derived from EFA and the summarization of latent constructs of students' Logical thinking aligned with Piaget's theory of cognitive development. Considering the response rate results to the indicators, the students have shown peak performance when using proportional reasoning. On the contrary, they need to perform better analogical and correlational reasoning. Also, students' combinatorial and inductive reasoning performance has been almost the same. These results can help design educational examples to teach and learn algebraic-geometric transformations and to identify and develop logical thinking structures, especially those related to mathematical reasoning. Moreover, the different performances of students in using reasoning strategies and adverse performance in the use of analogical reasoning strategies and correlational reasoning is a warning for authors and mathematics educators. Therefore, it requires more attention to the development of reasoning skills in the content of textbooks and teaching approaches in the field of algebraic-geometric transformations.

Due to the conditions of the Covid-19 epidemic, we collected most of the research data during hybrid training and in virtual and electronic form. 43 out of the 423 collected samples were discarded as outliers (see participants section). To generalize the findings, examining more samples in future research is suggested.

One of the requirements to implement logical thinking is the awareness of various mental factors and activities used when dealing with problems (Ab et al., 2019). On the other hand, mathematics courses have a unique evolutionary influence and provide a fundamental prerequisite for logical thinking development (Yunus, 2021). Hence, mathematics education policies and programs for developing skills, such as thinking and reasoning abilities, can identify latent constructs of students' logical thinking when dealing with different mathematical topics, such as algebraic-geometric transformations. Examining the effect of various transformation problems in developing each type of reasoning in students' logical thinking is suggested for future research.

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Authors' contribution

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Maryam Shahmohammadi. Elahe Aminifar, Modjtaba Ghorbani and Bahram Siyavashpour contributed in data analysis and discussing the results. The first draft of the manuscript was written by Maryam Shahmohammadi and Elahe Aminifar. All authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Competing Interests

The authors have no competing interests to declare that are relevant to the content of this manuscript and did not receive support from any organization for the submitted work.

Ethics approval

The consent was obtained from the parents of all individual participants in the study.

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Appendices

Table 1 *Communalities*

Indicator	Q_2	Q_4	Q_5	Q_6	Q_7	Q_{10}	Q_{11}	Q_{12}	Q_{14}	Q_{15}	Q_{16}	Q_{17}	Q_{18}	Q_{20}
Initial	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Extraction	0.51	0.99	0.58	0.58	0.98	0.61	0.99	0.65	0.56	0.53	0.56	0.58	0.69	0.74

Table 2 *The answers of the students to the loaded indicators in the latent factors*

Indicators	**Frequency percentage of answers**		
	True	**False**	**No answer**
Q_4	73.4	26.3	0.3
Q_7	76.3	23.2	0.5

Q_{11}	79.5	19.5	1
Q_{14}	87.3	11.6	1.1
Q_{15}	82.1	17.4	0.5
Q_{17}	88.7	10.0	1.3
Q_{10}	60.3	38.7	1.1
Q_{12}	55.8	42.1	2.1
Q_{18}	57.6	40.0	2.4
Q_{2}	78.9	20.3	0.8
Q_{5}	77.6	21.9	0.5
Q_{6}	73.1	26.1	0.8
Q_{16}	60.0	38.4	1.6
Q_{20}	58.7	39.7	1.6