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An Inventory Model to Control Deterioration Rate Using Preservation Technology with Non-Linear Holding Cost

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Abstract: - Objectives: The current work focuses on developing a mathematical model for minimizing inventory costs by examining a scenario involving a food product (i.e., vegetable) retail market, evaluating the outcome, and proposing solutions to unanswerable questions of existing models.

Methods: We create a mathematical model for retail market situations with deteriorating products and calculate the retailers' overall cost by using differential equations with initial and boundary conditions.

2. In particular, we analyze a non-linear function of price and stock to define the demand function. Control the rate of degradation in the current model by employing preservation technology as well.

3. A database inferred from previous research and some data derived from the vegetable farmhouse are used in this model. In order to illustrate the most and least important elements, we have included a sensitivity analysis and a numerical example with a graphical representation of the total cost function.

4. We created an algorithm to determine the ideal cost based on the Hessian matrix and the Calculus derivative method.

Findings: This model provides retailers with insights for effective stock management by using preservation technology while optimizing the cast of retailing deteriorating products with non-linear demand patterns that depend on stock and price. Sensitivity analysis showed that some factors that have the biggest impact on retailers, overall cost which are non-linear demand and holding cost parameters. The smaller optimal inventory cycle time and the large value of time at which a shortage happens raise the overall cost. Also, using preservation technology controls the deterioration rate and, left the least impact on overall cost. Additionally, a three-dimensional graph of cost minimization is offered to illustrate the Convexity of total cost.

Novelty: Implementation of price and stock-dependent demand and non-linear holding costs along with preservation technology and partial backlog shortages.

Keywords: Preservation technology investment, Stock and Price dependent demand, shortage with partial backlogging, Deterioration rate

1 Introduction:

In today's dynamic marketplace, effective inventory management stands as a cornerstone for sustainable business operations. In our day-to-day life, we deal with deteriorating items. It is very difficult to analyze Inventory management in case of deteriorating items. These types of items are divided into two categories: items that lose part or total value over time due to new technology, such as call phones, computer chips, fashion, and seasonal goods. A not

her type that is decomposed, degenerate, evaporative, or lapsed over time, such as meat, vegetables, drugs, fruits, Cosmetics, volatile liquids, and food packets. In this paper work on perishable item vegetables (e.g. Cauliflower, Mushroom, etc.). Apply the preservation to increase the life span keep the thing fresh and preserve quality. Therefore, suppliers spend some money on preservation technologies to protect goods, and researchers have investigated the efforts of preservation to keep products fresh so that customers can buy more things.

Demand is the most important factor to the majority of the businesses. Most businesses depend on the customer's demand. A high demand factor encourages an organization to about more production. It is observed that market demand directly affects the selling price and stock of the item. In short, the demand rate is higher by lowered sales price and enough availability of stock. Using a proper strategy improves the convenience of businesses in handling inventory systems, and in this mode considers a non-linear function of sales price and stock: Moreover, the holding cost is considered as a non-linear quadratic function of time. Conversely, shortages are often considered in many realistic inventory control schemes, where the demand can be backlogged until the order is re-filled in the scheme, or it may be skipped depending on the customer's preference and its products. Dye, C.Y. (2020, 2013), developed a non-instantaneous deteriorating Inventory model to control deterioration by using preservation technology. A time-dependent inventory model for objects with exponentially decreasing demand

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was developed by Ghare and Schrader (1963). A Standard Economic Order Quantity model was developed by F.W. Harris (1915) this model was solved by using two methods i.e. Tabulation Method and Algebraic Method. P.H. Hsu et al (2010) examined the preservation technology Investment for decay items with a constant demand rate and deterioration rate. S.K. Indrajitsinga et al. (2018) extended the crisp model Shama and Vijayo (2015) to a fuzzy model with uncertainties like demand, holding cost deterioration cost, etc. considered as fuzzy triangular number.

M.Y. Jani (2021) gave insight into the potential applications of credit financing and preservation technology for reducing the rate of deterioration and giving flexible financing options for retailers. Pal, H. et al. (2018) give an Inventory model to control deterioration which started after a random period and started after getting the delivery by the retailer. To decrease the deterioration using preservation technology. Saha, S. et al. (2017) invented a way to maximize retailer's profit by using dynamic retailer investments in green operations while considering the reference price effect. Singh, S.R. et al. (2016) developed an order quantity model with stock dependent. To control the deterioration retailers used preservation technology, and also used the applications of trade credit to control the total Cost. The authors Indrajitringha, S.K. et al. (2021) have a fuzzy model as well as a crisp model with stock-dependent demand and constant holding Cost. To control the deterioration of products the retailer used preservation technology. They suggested adopting a non-linear holding cost to meet the reality of inventory and could be an extension of their model. Few studies have also used preservation technology with stock-based, price-based demand, also using constant and linear holding cost in inventory models with backorder difficulties in the relevant literature that is currently available.

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The consequences of employing a preservation technology to Control deterioration with non-linear holding costs under a partial backlog event of shortage have thus not received much thought.

In light of this, the following query emerge

- How can an Inventory model be created that takes into account the non-linear holding costs and non-linear demand patterns i.e., stock and price dependent concerning problems with partial backlog shortages?
- what could be the relation between the different parameters and their impact on the total cost for the proposed inventory model for perishable product?
- what strategic measures can be taken by the retailer to minimize the cost for this Inventory scenario by using preservation technology in the case of perishable product?

This paper investigates the inventory problem for perishable items that have non-linear demand rates and holding costs. We attempted to respond to the above-mentioned queries in our newly constructed model. The present research study has been presented in the following manner: Section 1 contains an introduction as well as a review of prior research. Section 2 outlines the process. Section 3 contributes to the findings, and the discussion includes numerical examples, sensitivity analysis, and managerial implications. Section 4 concludes the study.

1.2 Literature Review

In this study, authors tested the optimal trade credit, preservation technology investment, and replenishment strategies that maximize the retailer's profit after the default risk occurs over a finite planning horizon. Yang, C.T. (2015) analyzed the optimal retail strategy by considering demand rate varies simultaneously with time and the length of credit period offered to the Customers.

Table 1: Comparison of the present work with the existing literature

Authors Name	Preservation Technology	Partial Backlog	Holding Cost	Demand rate	Solution Procedure
Das et. al. [1]	Yes	Yes	Constant	Selling price dependent	Soft Computing
Indrajitsingha et. al. [8]	Yes	Yes	Constant	Linear	Mathematically
Mishra et. al. [11]	Yes	No	Constant	Price dependent	Mathematically

Mishra et. al. [12]	No	No	Constant	Hybrid- price-stock dependent	Mathematically
Sindhuja et. al. [16]	Yes	Yes	Constant	Quality dependent	Mathematically
Sharma D. K et. al [15]	Yes	No	Constant	Linear	Mathematically
Adria´n Mac´ıas-Lo´pez et al. [10]	No	No	Non-linear	Price dependent	Mathematically
Present Paper	Yes	Yes	Non-linear	Price and Stock-dependent	Mathematically

2. Notation and Assumptions

2.1 Notations

The Notation is given below:

α: Initial demand rate.

β: Positive demand coefficient.

n(τ): Deterioration rate with investment in preservation technology.

n_o: Deterioration rate without investment in preservation technology.

η: Sensitive parameter of investment to the deterioration rate.

q(t): On hand stock at time t.

q: Initial Inventory level.

D(s, q(t)): Demand rate.

C_{hc}: Holding cost.

C_{sc}: Shortage cost.

C_{lc}: Lost sale cost

C_{dc}: Deterioration cost.

C_{oc}: Ordering cost.

C_{pc}: Purchasing cost.

T: The duration of the replenishment cycle.

$TAC(\tau,t_1)$: Total average cost for inventory management $0 \leq t_1 \leq T$.

2.2 Assumptions

The inventory model is based on the following assumptions:

i. The demand for the product depends on price and stock, for the price-dependent function is considered as linear.

ii. The inventory system involves only a single product.

iii. The lead time is assumed to be negligible. Shortages are allowed with partial backlogging and the backlogging rate is $e^{-\lambda(T-t)}$; λ is the backlogging variable and positive constant.

iv. It is assumed that $n(\tau) = n_o e^{-\eta\tau}$ and n(τ) is the rate of deterioration without investment in preservation technology and η is a sensitive parameter of investment to the deterioration rate.

v. The rate of replenishment is finite.

vi. The holding cost is non-linear and modeled with a quadratic function that depends on time i.e. $h+h_1t+h_2t^2$.

2.3Mathematical Formulations for the Inventory Model

In this section, we formulate a mathematical model for the inventory system. Products in the system are deteriorating at a constant rate and Preservation technology helps retailers to reduce the deterioration rate. Moreover, inventory level decreases due to product demand. Hence, the inventory level at any time t is governed by the differential equation given below.

Thus, the inventory system can be described by the following differential equations:

$$\frac{dq(t)}{dt}+n(\tau)q(t)=-D(S,q(t)) \;;0\le t\le t_1 \tag{1}$$

$$\frac{dq(t)}{dt}=-D(S,q(t))e^{-\lambda(T-t)} \;;t_1\le t\le T \tag{2}$$

With the boundary conditions

$$q(0)=q,\, q(t_1)=0 \tag{3}$$

The solutions of these equations are given by

$$q(t)=\frac{d(s)}{(n(\tau)+c_1)}\left[e^{(n(\tau)+c_1)(T_1-t)}-1\right];0\le t\le t_1 \tag{4}$$

$$q(t)=\frac{d(s)}{\lambda}\left[e^{-\lambda(T-t_1)}-e^{-\lambda(T-t)}\right];t_1\le t\le T \tag{5}$$

$$q=\frac{d(s)}{(n(\tau)+c_1)}\left[e^{(n(\tau)+c_1)T_1}-1\right] \tag{6}$$

By using the equation (4), (5) and (6) the value of parameters has been calculated

Now, the total cost is calculated by using the following basic costs:

1) **Ordering Cost**

The producer must place an order or multiple orders to purchase the raw materials. The ordering process may involve some expenditure like contacting the dealer, arranging meetings, and finalizing the orders, schedules, etc. So, the required cost for ordering is given by the following equation

$$OC = C_{oc} \tag{7}$$

2) **Holding cost**

The purchased raw materials are to be stored or held in some place before and after the process of production in the case of vegetables. There is a cost associated with this holding of raw and finished materials. Therefore, the holding cost is given by the following equation

HC= $$\frac{d(s)}{\left(n(\tau)+c_1\right)^2}\left[\begin{array}{l} h\left\{e^{\left(n(\tau)+c_1\right)t_1}-\left(n(\tau)+c_1\right)t_1-1\right\}+\frac{h_1}{\left(n(\tau)+c_1\right)}\left\{e^{\left(n(\tau)+c_1\right)t_1}-\left(n(\tau)+c_1\right)t_1-1\right\} \\ +\frac{h_2}{\left(n(\tau)+c_1\right)^2}\left\{2e^{\left(n(\tau)+c_1\right)t_1}-\left(n(\tau)+c_1\right)^2 t_1^2-2\left(n(\tau)+c_1\right)t_1-2\right\} \\ -\left(n(\tau)+c_1\right)\left\{\frac{t_1^2 h_1}{2}-\frac{t_1^3 h_2}{3}\right\} \end{array}\right]$$

(8)

3) **Purchasing cost**

The raw materials should be purchased to make the finished products. There are some costs associated with the purchase of the raw materials for the production of the finished products. The following equation gives the purchasing cost of the raw materials

PC = $C_{pc}q$

$$= C_{pc}\frac{d(s)}{\left(n(\tau)+c_1\right)}\left[e^{\left(n(\tau)+c_1\right)t_1}-1\right] \tag{9}$$

4) **Shortage cost**

There might be a shortage of raw materials when ordered and purchased for making the finished products. The producer must pay an additional cost to make up for the disruption caused by the shortage. Therefore, the shortage cost is given by the following equation

$$\text{SC} = -C_{sc}\frac{d(s)}{\lambda^2}\left[\lambda\left(T-t_1\right)e^{-\lambda(T-t_1)}-1+e^{-\lambda(T-t_1)}\right] \tag{10}$$

5) **Lost Sale cost**

If any organization fails to satisfy the Customer's demand, it provokes a cost known as the cost of lost sales which is given by the following equation

$$\text{LSC} = -C_{lc}\frac{d(s)}{\lambda}\left[\lambda\left(T-t_1\right)-1+e^{-\lambda(T-t_1)}\right] \tag{11}$$

6) Deterioration cost

All of the produced finished products are not ready for sending to the markets. Some parts of the products undergo deterioration over time. So, the producer has to pay a significant amount to manage and make for the undesirable situation. Therefore, the deterioration cost is given by the following equation

$$\text{DC} = C_{dc}\frac{d(s)n(\tau)}{\left(n(\tau)+c_1\right)^2}\left[e^{\left(n(\tau)+c_1\right)t_1}-\left(n(\tau)+c_1\right)t_1-1\right] \tag{12}$$

7) Preservation technology cost

To minimize the deterioration of the products, the producer invests in some advanced preservation technology. That would require a significant expense to install the technology. The following equation gives the preservation technology cost

$$\text{PTC} = \tau T \tag{13}$$

Average total cost TAC(τ,t_1) for the period T

$$TAC(\tau,t_1) = \frac{1}{T}[HC+DC+OC+PC+SC+LSC+PTC] \quad (14)$$

$$= \frac{1}{T}\left\{\begin{array}{l} C_{oc} + \dfrac{d(s)}{(n(\tau)+c_1)}\left[\begin{array}{l} h\left\{e^{(n(\tau)+c_1)t_1} - (n(\tau)+c_1)t_1 - 1\right\} + \dfrac{h_1}{(n(\tau)+c_1)^2}\left\{e^{(n(\tau)+c_1)t_1} - (n(\tau)+c_1)t_1 - 1\right\} \\ + \dfrac{h_2}{(n(\tau)+c_1)^3}\left\{2e^{(n(\tau)+c_1)t_1} - (n(\tau)+c_1)^2 t_1^2 - 2(n(\tau)+c_1)t_1 - 2\right\} - \left\{\dfrac{t_1^2 h_1}{2} - \dfrac{t_1^3 h_2}{3}\right\} \end{array}\right] \\ + C_{dc}\dfrac{d(s)n(\tau)}{(n(\tau)+c_1)^2}\left[e^{(n(\tau)+c_1)t_1} - (n(\tau)+c_1)t_1 - 1\right] + C_{pc}\dfrac{d(s)}{(n(\tau)+c_1)}\left[e^{(n(\tau)+c_1)t_1} - 1\right] \\ - C_{sc}\dfrac{d(s)}{\lambda^2}\left[\lambda(T-t_1)e^{-\lambda(T-t_1)} - 1 + e^{-\lambda(T-t_1)}\right] - C_{lc}\dfrac{d(s)}{\lambda}\left[\lambda(T-t_1) - 1 + e^{-\lambda(T-t_1)}\right] + \tau T \end{array}\right\} \quad (15)$$

For a small x value, the Taylor series says that the exponential function has an approximation of $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ using this result in the above equation.

$$TAC(\tau,t_1) = \frac{1}{T}\left[\begin{array}{l} (C_{oc} + \tau T) + \\ d(s)\left\{\begin{array}{l} \left(\left(h + \dfrac{h_1}{(n(\tau)+c_1)}\right)\left(\dfrac{t_1^2}{2} + (n(\tau)+c_1)\dfrac{t_1^3}{6}\right) + h_2\dfrac{t_1^3}{6} - \dfrac{1}{(n(\tau)+c_1)}\left(h_1\dfrac{t_1^2}{2} + h_2\dfrac{t_1^3}{6}\right)\right) \\ + C_{dc} n(\tau)\left(\dfrac{t_1^2}{2} + (n(\tau)+c_1)\dfrac{t_1^3}{6}\right) + C_{pc}(n(\tau)+c_1)\left(\dfrac{3}{2}t_1^2 + \dfrac{t_1^3}{6}\right) \\ - C_{sc}(T-t_1)^2\left(-\dfrac{1}{2} + \dfrac{\lambda}{3}(T-t_1) - \dfrac{\lambda^2}{6}(T-t_1)^2\right) - C_{lc}\dfrac{\lambda}{2}(T-t_1)^2\left(1 - \dfrac{\lambda}{3}(T-t_1)\right) \end{array}\right\} \end{array}\right] \quad (16)$$

Where $d(s) = \alpha - \beta s, n(\tau) = n_o e^{-\eta\tau}$

$$\frac{\partial TAC(\tau,t_1)}{\partial \tau} = \frac{1}{T}\left[d(s)\left\{\begin{array}{l} -\dfrac{n'(\tau)h_1}{(n(\tau)+c_1)^2}\left(\dfrac{t_1^2}{2} + (n(\tau)+c_1)\dfrac{t_1^3}{6}\right) + \left(h + \dfrac{h_1}{(n(\tau)+c_1)}\right)\left(n'(\tau)\dfrac{t_1^3}{6}\right) \\ + \dfrac{n'(\tau)}{(n(\tau)+c_1)^2}\left(h_1\dfrac{t_1^2}{2} + h_2\dfrac{t_1^3}{6}\right) + C_{pc}n'(\tau)\left(\dfrac{3}{2}t_1^2 + \dfrac{t_1^3}{6}\right) \\ + C_{dc}\left(n'(\tau)\left(\dfrac{t_1^2}{2} + (n(\tau)+c_1)\dfrac{t_1^3}{6}\right) + n(\tau)n'(\tau)\dfrac{t_1^3}{6}\right) \end{array}\right\}\right] \quad (17)$$

$$\frac{\partial TAC(\tau,t_1)}{\partial t_1}=\frac{1}{T}\left[d(s)\left\{\begin{array}{l}\left(\left(h+\frac{h_1}{(n(\tau)+c_1)}\right)\left(t_1+(n(\tau)+c_1)\frac{t_1^2}{2}\right)+h_2\frac{t_1^2}{2}-\frac{1}{(n(\tau)+c_1)}\left(h_1t_1+h_2\frac{t_1^2}{2}\right)\right)\\+C_{dc}n(\tau)\left(t_1+(n(\tau)+c_1)\frac{t_1^2}{2}\right)+C_{pc}(n(\tau)+c_1)\left(3t_1+\frac{t_1^2}{2}\right)\\-C_{sc}(T-t_1)\left(-1+\frac{\lambda}{3}(T-t_1)+\frac{2\lambda^2}{3}(T-t_1)^2\right)+C_{lc}\lambda(T-t_1)\left(1-\frac{\lambda}{2}(T-t_1)\right)\end{array}\right\}\right] \quad (18)$$

Where $d(s)=\alpha-\beta s, n(\tau)=n_o e^{-\eta\tau}$

2.4. Solution methodology for finding the optimal solution

The non-linear optimization problem is followed as

minimize$TAC(\tau,t_1)$ subject to the conditions

$TAC(\tau, t_1) > 0$, $\tau > 0$, and $t_1 > 0$

Following necessary conditions to minimize the average total cost function $TAC(\tau,T_1)$ per unit time, the value of τ and t_1 that minimizes average cost can be obtained by solving the equation

$$\frac{\partial TAC(\tau,t_1)}{\partial \tau}=0 \quad and \quad \frac{\partial TAC(\tau,t_1)}{\partial t_1}=0 \quad (19)$$

Satisfying the conditions

$$\frac{\partial^2 TAC(\tau,t_1)}{\partial \tau^2}>0 \quad , \quad \frac{\partial^2 TAC(\tau,t_1)}{\partial t_1^2}>0, \ and$$

$$\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial \tau^2}\right)\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial t_1^2}\right)-\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial \tau\partial t_1}\right)^2>0 \quad (20)$$

Algorithm for finding the optimal solution

To solve the problem, the mathematical software MATHEMATICA 11.0 is used to find the optimal solution. Also, at the time for solving optimal solutions, reject a higher degree than 2 in Taylor's series.

Step 1: Place the parameters α, β, s, h, h_1, h_2 etc.

Step 2: Solve equation (19) to find out the optimal values τ, t_1.

Step 3: If all the conditions given in equation (20) are satisfied then get the optimal values.

Step 4: Place the particular values (τ, t_1).

Step 5: Place the parameters α, β, s, h, h_1, h_2, etc along with the values τ and t1 in equation (15).

Step 6: Find out the total average cost TAC (τ, t1).

Step 7: Construct the optimal total average cost TAC* (τ, t1).

Step 8: Evaluate the optimal total cost TAC* (τ, t_1) and corresponding optimal value of $\tau = \tau^*$, $t_1 = t_1^*$, and q = q*.

Step 9: Stop.

3. Results and Discussion

3.1Numerical Example

In this section, consider a real scenario taking some data from vegetables from a house and using some secondary data to prove the authenticity of the model.

Example

Consider a real situation in which the demand the demand of the product depends on price and stock, for the price-dependent function is considered linear as

$$D(s,q(t)) = \begin{Bmatrix} d(s)+c_1q(t) & ;0 \le t \le t_1 \\ d(s) & ;t_1 \le t \le T \end{Bmatrix}; Where\ d(s) = (\alpha - \beta s)$$

$\alpha = 1000$; $\beta = 10$; $s = 30$; $T = 13$; $\eta = 0.4$; $\lambda = 0.04$; $n_o = 4$; $h = 1.75$; $h_1 = 0.15$; $h_2 = 0.25$; $C_{pc} = 10$; $C_{oc} = 250$; $C_{sc} = 10$; $C_{dc} = 4$; $C_{lc} = 9$; $c_1 = 0.5$;

By using MATHEMATICA, calculate the optimal solution for the inventory system.

$$\frac{\partial TAC(\tau,t_1)}{\partial \tau} = 0 \quad and \quad \frac{\partial TAC(\tau,t_1)}{\partial t_1} = 0$$

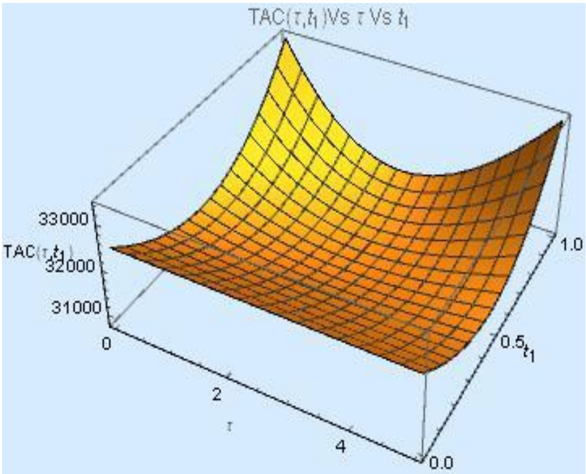
Get the optimal solution

$$\tau^* = 2.37803, t_1^* = 0.712086, q^* = 942.988, and\, TAC^* = 30785.9$$

Additionally, this solution satisfies all conditions for optimality:

$$\frac{\partial^2 TAC(\tau,t_1)}{\partial \tau^2} = 338.293 > 0 \ , \ \frac{\partial^2 TAC(\tau,t_1)}{\partial t_1^2} = 7769.27 > 0, \ and$$

$$\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial \tau^2}\right)\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial t_1^2}\right) - \left(\frac{\partial^2 TAC(\tau,t_1)}{\partial \tau \partial t_1}\right)^2 = 2.61163 \times 10^6 > 0$$



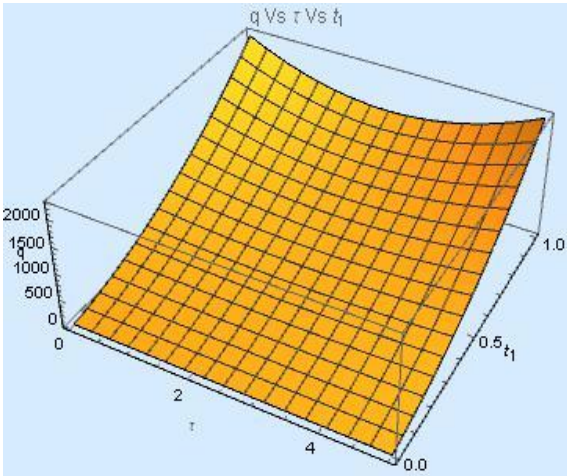


Fig.2(a) **Fig. 2 (b)**

Graphical representation of Covex optimization of Total cost function with Preservation Cost τ shortage period t_1.

Graphical representation of the Ordered quantity of profit function with Preservation Cost τ and and shortage period t_1.

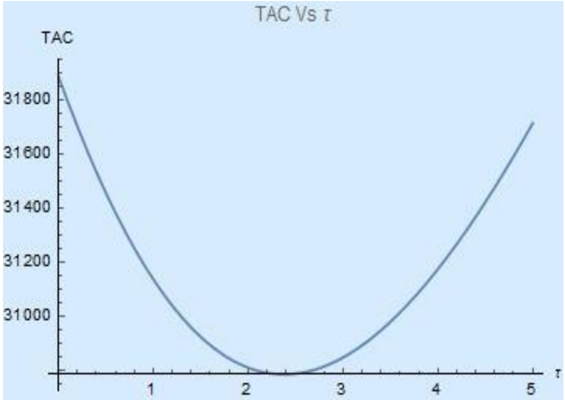


Fig. 2 (c) Two dimension representation of TAC versus τ

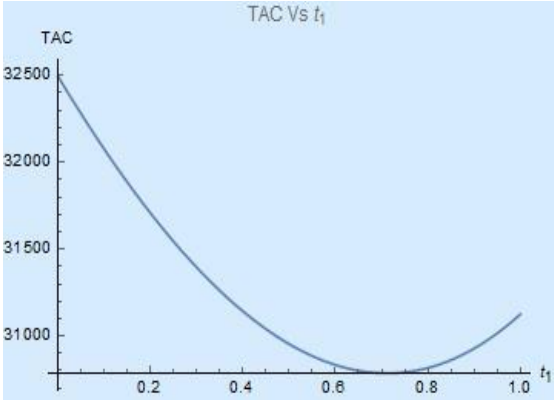


Fig.2 (d) Two dimension representation of TAC versus t_1

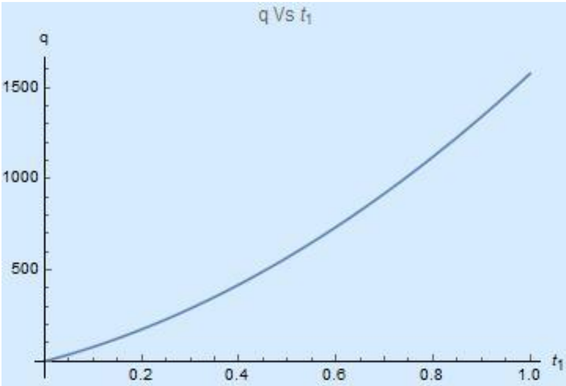


Fig. 2 (e) Two dimension representation of q versus t_1

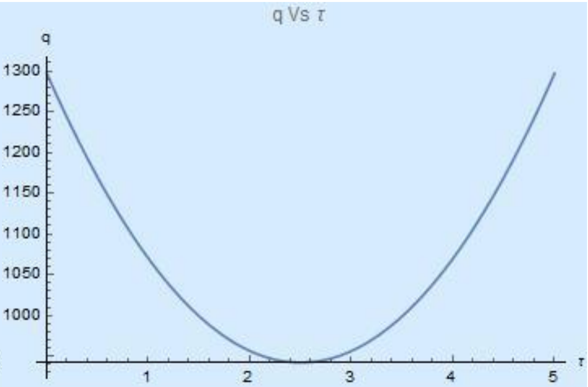


Fig. 2(f) Two dimension representation of q versus τ

3.2 Sensitivity Analysis

The managerial implications discussed in this section are based on sensitivity analysis of various input factors to total average cost and decision variables. we look into how changes in parameter values affect the ideal values of τ, t, q, and total average Cost By adjusting each parameter's value in stages of (-50%, -25%, +25%, +50%), the sensitivity analysis is displayed. Results are shown in table 2. The following observation can be made from the results of Table 2

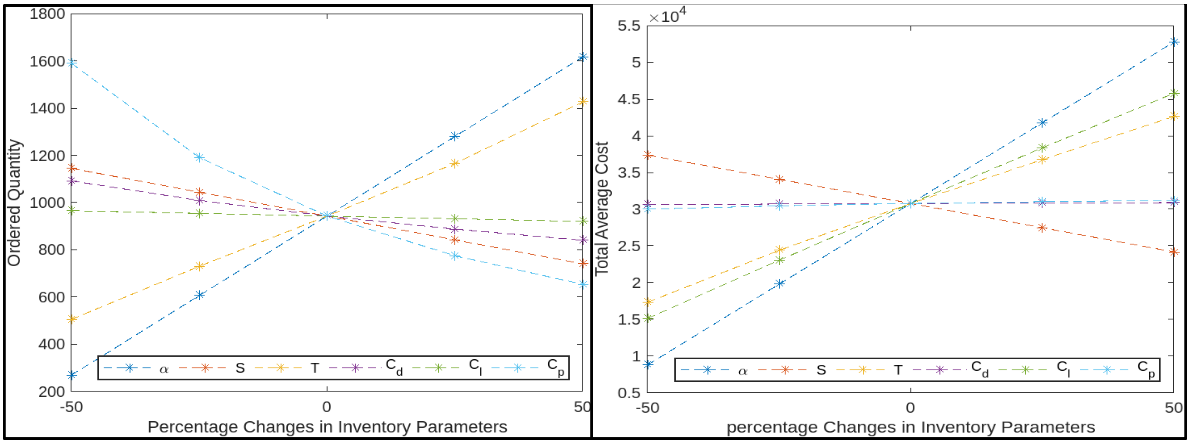
Parameter	Values	%	Preservation cost	Shortage Time	Total Average cost	Ordered Quantity
	500	-50%	2.3706	0.712205	8811.4	269.522
	750	-25%	2.37637	0.712113	19798.6	606.256
α	**1000**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	1250	25%	2.37881	0.712073	41773.1	1279.72
	1500	50%	2.37927	0.712065	52760.4	1616.45
	5	-50%	2.37855	0.712077	37378.2	1145.03
	7.5	-25%	2.37831	0.712081	34082.1	1044.01
β	**10**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	12.5	25%	2.37767	0.712092	27489.7	841.987
	15	50%	2.37722	0.712099	24193.5	740.949
	15	-50%	2.37855	0.712077	37378.2	1145.03

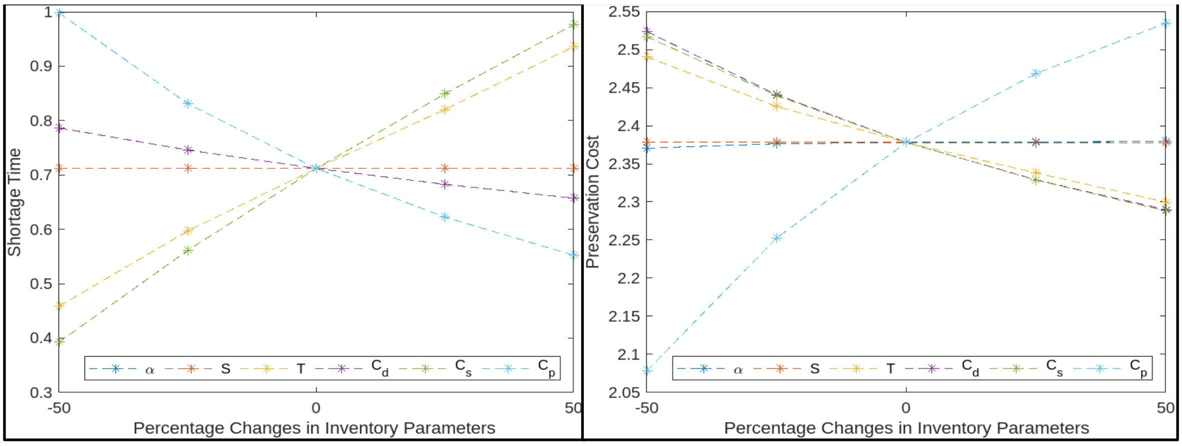
	22.5	-25%	2.37831	0.712081	34082.1	1044.01
S	**30**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	37.5	25%	2.37767	0.712092	27489.7	841.968
	45	50%	2.37722	0.712099	24193.5	740.949
	6.5	-50%	2.49029	0.458876	17328.6	505.462
	9.75	-25%	2.42535	0.597168	24411.2	730.274
T	**13**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	16.25	25%	2.33803	0.82064	36751.7	1165.7
	19.5	50%	2.29975	0.936251	42629	1426.31
	0.25	-50%	2.2431	0.748728	30691.7	969.722
	0.375	-25%	2.31053	0.729998	30739.9	956.112
C_1	**0.5**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	0.625	25%	2.44558	0.694941	30829.9	930.323
	0.75	50%	2.5132	0.678516	30871.9	918.091
	0.2	-50%	4.75011	0.712135	30788.3	1039.75
	0.3	-25%	3.16938	0.712102	30786.7	971.297
η	**0.4**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	0.5	25%	-------	--------	-------	954.824
	0.6	50%	1.58601	0.712069	30785.1	1006.8
	2	-50%	2.70244	1.16942	29679.8	765.092
	3	-25%	2.4871	0.885378	30367.3	854.04
n_o	**4**	**0**	**2.37803**	**0.712086**	**31067.1**	**942.988**
	5	25%	2.31193	0.595467	31193.6	1031.94
	6	50%	2.70097	0.550039	30785.9	1120.88
	2	-50%	2.52361	0.786534	30627.3	1091.92
	3	-25%	2.44073	0.745957	30712.8	1009.28
C_d	**4**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	5	25%	2.32898	0.68297	30849.8	887.748
	6	50%	2.28967	0.657426	30906.7	840.523
	125	-50%	2.37803	0.712086	30776.3	942.988
	187.5	-25%	2.37803	0.712086	30781.1	942.988
C_o	**250**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	312.5	25%	2.37803	0.712086	30790.7	942.988
	375	50%	2.37803	0.712086	30795.5	942.988
	5	-50%	2.517	0.392954	15141.1	410.184
	7.5	-25%	2.43957	0.561432	23052.4	668.937
C_s	**10**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	12.5	25%	2.32869	0.849518	38360.2	1228.51
	15	50%	2.28811	0.976582	45789.4	1522.9
	0.075	-50%	2.37798	0.712218	30785.7	943.247
	0.1125	-25%	2.378	0.712252	30785.8	943.118
h_1	**0.15**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	0.1875	25%	2.37805	0.71202	30786	942.858
	0.225	50%	2.37808	0.711953	30786.1	942.729
	0.125	-50%	2.89407	0.87882	30490.7	1304.39
	0.1875	-25%	2.37793	0.712146	30785.8	943.107

h_2	**0.25**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	0.3125	25%	2.37812	0.712025	30786	942.869
	0.375	50%	2.37821	0.711965	30786.1	942.749
	0.875	-50%	2.37314	0.720337	30766.8	959.196
	1.3125	-25%	2.3756	0.716181	30776.4	951.016
h	**1.75**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	2.1875	25%	2.38042	0.708052	30795.2	935.109
	2.625	50%	2.38278	0.704076	30804.5	927.374
	0.02	-50%	2.33718	0.824627	35491.7	1174.27
	0.03	-25%	2.35988	0.760678	32935.5	1040.05
λ	**0.04**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	0.05	25%	2.39015	0.680776	29062.4	882.693
	0.06	50%	2.39522	0.667914	27778.1	858.434
	4.5	-50%	2.37362	0.72368	31397.2	965.761
	6.75	-25%	2.37582	0.717893	31091.7	954.365
C_l	**9**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	11.25	25%	2.38026	0.706257	30479.8	931.631
	13.5	50%	2.38251	0.700407	30173.5	920.293
	5	-50%	2.07851	0.997907	30011.3	1589.69
	7.5	-25%	2.2527	0.831199	30469.2	1191.1
C_p	**10**	**0**	**2.37803**	**0.712086**	**30785.9**	**942.988**
	12.5	25%	2.46848	0.622507	31018.3	774.874
	15	50%	2.53471	0.552585	31196.3	654.032

In an inventory system, a retailer should know the impact of the parameters of the system on the average total cost function. It should be evident to the retailer when one can get the minimum expenditure irrespective of the increase or decrease of parameters. This present model illustrates the relevance of the model. It also locates the important impacts of vegetable farmhouses; we will go through the sensitivity analysis concerning the different parameters.

- As the values of parameters α, T, n_o, C_s, h_2 rise, the total average cost rises, but the total average cost falls as the values of β, S, λ, and C_l rise. Changes in α, T, C_s, λ, and βhave a significant impact on total average cost. Changes in n_o, h_2, S, and C_l affect it less, and changes in C_1, C_d, C_o, and C_p affect it very little. There is no change in total average cost i.e. constant while parameters η, h_1, and h rise.





Managerial Implications

(a) It is possible to conclude that the optimal preservation cost τ^*, the optimal ordered quantity Q^*, and the optimal total average cost per unit time (TAC^*) fall when the value of some parameters 'β' and 'S' rises and the values of the other parameters remain stable. This shows that as the market demand rate needs to increase in response to an increase in 'β' and 'S', the organization will need to decrease the replenishment cycle and raise the order amount per cycle to meet the growing market demand. To minimize the total average cost, the company will also need to reduce costs like holding costs, purchasing costs, etc. To sell more products and slow down the rate of product deterioration, the Company will invest more money in improving preservation technology.

(b) It is observed that most of the time optimal ordered quantities decrease while optimal total average cost rises, but the optimal preservation cost falls. It implies that the Company will reduce the shortage time and increase the ordered quantity then get the minimum total average cost.

4. Conclusion: - In this present paper, our new model on mathematical inventory has been developed and presented, for deteriorating items with non-linear Holding costs, and preservation technology, with partial shortage due to a sudden increase in demand. In day-to-day issues, it has been noticed that public demand plays an important role in the case of purchasing perishables like vegetables, fruits, etc. In the case of vegetables deterioration rate plays a vital role for the retailer to maintain the life span of the product. So, to reduce the deterioration, the retailer uses preservation technology to fulfil the demand of the consumers. Keeping this in mind, there is a link between demand, the stock level, and price. On evaluating the model, we found that this model helps the retailer of the company to minimize the total costs in terms of holding cost, shortage, etc by reducing order size or cycle time. This model also helps the retailer to minimize costs and gain profit by reducing deterioration costs with the help of preservation technology.

Future research can be extended to credit policy with advanced payment, inflation, and fuzzy environment to address the uncertainty of demand and preservation technology to reduce the deterioration rate, as well as using soft computing techniques to find the optimum solution.

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