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Direction of Arrival Estimation Using Deep Learning for V-Shape Coprime Array with Low SNR

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Abstract: *- Direction of arrival is the most important issue in the array signal processing the use of traditional technique for the DOA estimation would require large number of array sensor this lead to unfeasible and in the same time very costly .so the use of coprime array will use less number of array with the same result of the use of traditional method the cooperation of this method with the deep learning method that suggested in this paper will provide efficient DOA estimation .the suggested scenario is to use V-shaped coprime array incorporates a sensor. For instance, at the same number used in the previous model with M = 5 and N = 4, we have a virtual sensor of 27 to solve sources, which equates to 20 sources. In terms of coprime array design, the VCA outperforms the CPA. However, the deep learning model design for estimation of the DOA uses CNN with prior knowledge of the number of sources, and it compares with other techniques like MUSIC, ROOTMUSIC, and the robust ℓ2,1-SVD method. The proposed CNN's RMSE is lower than that of Music and Root-MUSIC.*

Keywords: DOA estimation, MUSIC, deep learning, 2D V-shape coprime arrays, convolutional neural networks

I. Introduction

The direction-of-arrival (DOA) estimation is a critical issue in array signal processing for applications such as radar, sonar, and wireless communications.[1] The MUSIC algorithm, also known as Multiple Signal Classification, is an effective technique due to its simplicity and ability to achieve performance that approaches theoretical limits. However, the performance limit of the MUSIC algorithm is to predict up to K (M -1) source directions for an M-element sensor array, as it requires at least one-dimensional noise subspace[2]. Uniform array structures are typically used in most applications, but nonuniform ones have gained significant attention due to their efficiency in terms of the number of components and their ability to provide underdetermined source estimates in situations when there are many sources.[3] The underdetermined circumstance is not effectively addressed by a nonuniform array structure. Minimum redundancy arrays (MRAs) are proposed as a solution, which typically provide less degrees of freedom (DOF) compared to this particular system, giving a greater range of DOF[4]. Nested array structures are introduced as a method for determining O(M^2), where the number of sensors is less than the number of sensors.[5] Coprime array structures are implemented to reduce coupling and allow a maximum of $K \leq MN$ sources to be used with a 1-D coprime array[6]. The degree of freedom (DOF) for the coprime planer array (CPA) using the modified generalized control performance assessment (GCPA) is equal to the sum of the product of N1M1 and N2M2 sensors. 2-D Direction of Arrival (DOA) estimates can be simplified by using L-shaped arrays instead of planar arrays[7]. Matrix estimation is performed to estimate the azimuth and elevation angles individually. The proposed method resolves up to MN sources using 4M+2N-3 sensors. It is the first to use deep learning for DOA estimation using VCA and attains accurate high-resolution Direction of Arrival (DOA) estimation using a limited number of sensors. In conclusion, the proposed method resolves up to MN sources using 4M+2N-3 sensors, is the first to use deep learning for DOA estimation using VCA, and achieves accurate high-resolution DOA estimation using a limited number of sensors. Deep Learning, often known as DL, is a relatively new technique for estimating DOA that has a number of benefits over other optimization-based techniques[8]. approaches that are based on deep learning do not need any optimization after training, which results in straightforward operations such as addition and multiplication. Additionally, these approaches do not call for any particular parameter adjustment[9]. Deep neural networks (DNNs), multilayer autoencoders, and deep Convolutional Neural Networks (CNNs) are some of the approaches that are discussed in the work. These techniques are used for estimating the DOA in an array of applications.[10] The signal covariance matrix has been

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used by DNNs for the purpose of DOA classification of two targets; nevertheless, the performance of these neural networks has been challenged due to the fact that they provide poor DOA estimate results in high SNR[11]. There was also a proposal for a multilayer autoencoder that included a number of parallel multilayer classifiers. This design, known as a multi-layer perceptron (MLP), was also considered; however, owing to the use of 1-dimensional filters, it did not demonstrate any substantial performance improvements in terms of DOA estimates. The work investigates the problem of narrow-band DOA estimate, with a particular emphasis on the estimation of the directions along which the signal is heading. at addition to this, they present a DL-based technique for estimating pseudo-spectra, which was shown at high SNR and assumed that the number of sources was already known[12]. A deep neural network (DNN) for beamforming using a single-snapshot (SCM) was presented in the field of acoustics; however, this technique cannot be used in situations with low signal-to-noise ratio (SNR), since the number of snapshots and sensors for these scenarios has to be significantly increased. According to our best knowledge, there is no particular DL-based approach that has been developed for the purpose of providing reliable DOA estimate in the low SNR regime. One further drawback of these approaches is that they are trained for a certain number of pictures, which results in large variances for varied quantities of snapshots and different SNR values.

The paper has been divided into five parts. Section 2 covers the relevant mathematical foundations related to the issue. Section 3 discusses the modeling of Convolutional Neural Networks for Direction of Arrival estimation. Section 4 includes comparisons and simulations. Section 5 offers a summary of the complete paper.

II. Preliminaries

AV-shaped array consisting of two sections positioned in the yz-plane, as seen in Figure 3.9a.To simplify, the sensors are positioned on axes referred to as $\mathbb{O}$- and $\mathbb{G}$-axes. The $\mathbb{O}$ -axis is designated for sensors that have their location established.

$$\mathbb{O} = \{y_i, z_i: -y_i \sin(\Omega/2) + z_i \cos(\Omega/2) = o_i\} \quad (1)$$

$$\mathbb{G} = \left\{y_i, z_i: -y_i \sin\left(\frac{\Omega}{2}\right) + z_i \cos\left(\frac{\Omega}{2}\right) = g_i\right\} \quad (2)$$

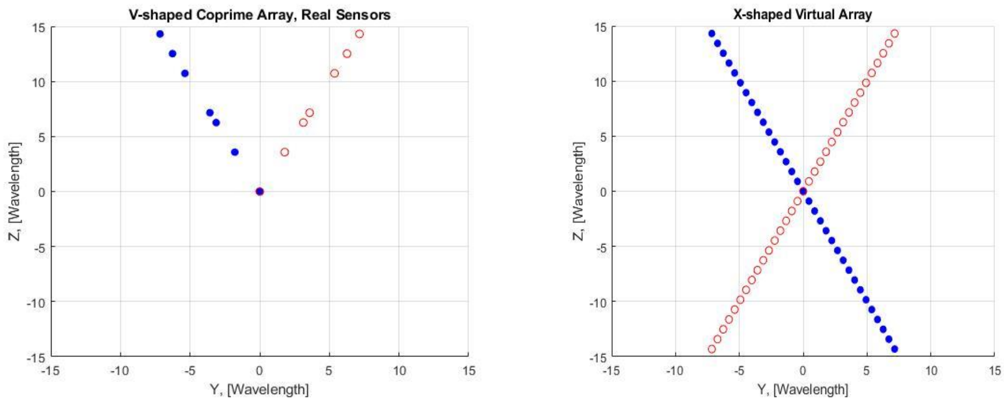

Figure 1: V-shaped coprime array (VCA) structure for M = 4, N = 7 and d $\lambda\backslash 2$. (a) The real sensor positions. (b) The contagious part of each co-array.

where$y_i$and z_i represent the coordinates of the sensors in a Cartesian coordinate system, and $\mathcal{G}$ represents the angle between two sections.$\mathcal{G}$-axis. It should be noted that o_i and $g_i$are integers that increase the size of the sparse array and create a bigger virtual array with coprime property and Vandermonde model. The part is divided into two subarrays, one with 2M items and the other with N elements. Here, M is less than N, and both M and N are coprime integers more than or equal to $\mathbb{N}^+$. The locations of the 2M sensors are represented by the set $\mathbb{S}_{2M}$ ={Nmd: $0 \leq m \leq 2M-1$}, while the locations of the N sensors are represented by the set $\mathbb{S}_N$={ Mnd: $0 \leq n \leq N-1$}Here, d represents the fundamental element spacing in the array, and for narrowband source signals, d $\lambda\backslash 2$ is used to prevent spatial aliasing . Hence, each section has a total of 2M + N - 1 sensors, and the whole array consists of 4M + 2N - 3 sensors. Let's assume that there are K source signals coming towards the array from different directions, denoted as $\Theta_k = \{\theta_k, \phi_k\}_{k=1}^{K}$represents the elevation angle θ_k and$\phi_k$represents the azimuth angle of the k^{th} source
The outputs of each section are determined by

$$\mathcal{O}(t_i) = \sum_{k=1}^{K} \boldsymbol{a}_{\mathcal{o}}(\Theta_k)s_k(t_i) + \boldsymbol{n}_{\mathcal{o}}(t_i)$$
$$\mathcal{G}(t_i) = \sum_{k=1}^{K} \boldsymbol{a}_{\mathcal{g}}(\Theta_k)s_k(t_i) + \boldsymbol{n}_{\mathcal{g}}(t_i) \quad (3)$$

Where $i = 1, \dots, T$ and T is the number of snapshots$\boldsymbol{n}_{\mathcal{o}}(t_i)$ *and* $\boldsymbol{n}_{\mathcal{g}}(t_i)$is white noise vector and $\{s_k(t_i)\}_{k=1,i=1}^{K}$ the source uncorrelated signal and the steering vector for the azimuth and elevation given by

$$[\boldsymbol{a}_{\mathcal{o}}(\Theta_k)]_i = \exp\left\{j\frac{2\pi}{\lambda}\mathcal{O}_i[-\sin(\phi_k) + \sin(\theta_k)]\right\}$$
$$[\boldsymbol{a}_{\mathcal{g}}(\Theta_k)]_i = \exp\left\{j\frac{2\pi}{\lambda}\mathcal{G}_i[\sin(\phi_k) + \sin(\theta_k)]\right\} \quad (4)$$

Where $\mathcal{o}_i$and $\mathcal{g}_i$ is the sensor position in in $\mathcal{O}$- and $\mathcal{G}$-axes respectively , $\mathcal{o}_i$ *and* $\mathcal{g}_i \in \mathbb{S}$ while $\mathbb{S} = \mathbb{S}_N \cup \mathbb{S}_{2M}$

The array covariance matrix for (3.39) is given as follow

$$\boldsymbol{R}_{\mathcal{O}} = E\{\mathcal{O}(t)\mathcal{O}^H(t)\} = \boldsymbol{A}_{\mathcal{O}}\boldsymbol{R}_{\boldsymbol{S}}\boldsymbol{A}_{\mathcal{O}}^H + \sigma_n^2\boldsymbol{I},$$
$$\boldsymbol{R}_{\mathcal{G}} = E\{\mathcal{G}(t)\mathcal{G}^H(t)\} = \boldsymbol{A}_{\mathcal{G}}\boldsymbol{R}_{\boldsymbol{S}}\boldsymbol{A}_{\mathcal{G}}^H + \sigma_n^2\boldsymbol{I} \quad (5)$$

The process of designing a V-shaped array entails identifying the sites of sensors that possess the coprime sampling property, as well as finding the V-angle, denoted as Ω. The parameter Ω governs the relationship between the estimate of azimuth and elevation angles, enabling independent calculation of the direction of arrival (DOA). In order to do this, it is necessary for the cross terms of the Fisher information matrix to be equal to zero. In order to do this, sensors are positioned based on the chosen V-angle. $\Omega = 2\tan^{-1}\left\{\sqrt{\frac{\bar{M}^2+3}{4M^2}}\right\}$ where M is the number of sensors in the virtual V-shaped array.

A. DOA Estimation for V-Shape Coprime Array

To generate the paired Direction of Arrival (DOA) estimations, the cross-covariance of $\mathcal{O}(t_i)$ and $\mathcal{G}(t_i)$ is calculated. In the following discussion, we first address the estimate of the azimuth angles only based on the use of $\mathcal{O}(t_i)$. Next, the elevation angles are calculated and automatically matched with the predicted azimuth angles.

1) Azimuth Angle Estimation

To estimate the DOA using MUSIC algorithm use the following formula

$$P_{\mathcal{O}}(\varphi) = \frac{1}{\mathbf{a}_{\mathcal{O}}^H(\varphi)\mathbf{E}_{\mathcal{O}n}\mathbf{E}_{\mathcal{O}n}^H\mathbf{a}_{\mathcal{O}}(\varphi)}$$
$$P_{\mathcal{G}}(\vartheta) = \frac{1}{\mathbf{a}_{\mathcal{G}}^H(\vartheta)\mathbf{E}_{\mathcal{G}n}\mathbf{E}_{\mathcal{G}n}^H\mathbf{a}_{\mathcal{G}}(\vartheta)} \quad (6)$$

Where φ and ϑ represent the angles of the Direction of Arrival (DOA), which are defined as

$$\varphi = -\sin(\phi)\sin\left(\frac{\Omega}{2}\right) + \sin(\theta)\cos\left(\frac{\Omega}{2}\right)$$
$$\vartheta = \sin(\phi)\sin\left(\frac{\Omega}{2}\right) + \sin(\theta)\cos\left(\frac{\Omega}{2}\right) \quad (7)$$

2) Elevation Angle Estimation

To calculate the elevation angles, the cross-covariance matrix is constructed.

$$\boldsymbol{\mathcal{R}}_{\mathcal{O}\mathcal{G}} = E\{\mathcal{O}(t)\mathcal{G}^H(t)\} = \mathbf{A}_{\mathcal{O}}\mathbf{R}_{S}\mathbf{A}_{\mathcal{G}}^H \quad (8)$$

Where the noise terms have been eliminated because of the assumption that the noise is spatially white. It is important to observe that in practical applications, the sample cross-covariance matrix $\mathcal{R}_{\mathcal{o}\mathcal{g}} = \frac{1}{T}\sum_{i=1}^{T}\mathcal{O}(t_i)\mathcal{G}^H(t_i)$ exists. The options are now accessible and the levels of loudness are negligible. Our objective is to calculate the steering matrix $A_{\mathcal{G}}$, where each column represents the elevation angles that are associated with the columns of the predicted steering matrix$\hat{A}_{\mathcal{G}}$. Given that the columns of $A_{\mathcal{O}}$ and $A_{\mathcal{G}}$ have same order, this procedure will result in automatically matched estimations of azimuth and elevation angles. Therefore, we want to solve the subsequent least squares issue, that is,

$$\hat{\mathbf{A}}_{\mathcal{G}} = \arg\min_{\mathbf{A}_{\mathcal{g}}} \left\|\mathbf{R}_{\mathcal{O}\mathcal{G}} - \hat{\mathbf{A}}_{\mathcal{O}}\mathbf{R}_{S}\mathbf{A}_{\mathcal{G}}^H\right\|_F^2 \quad (9)$$

The knowledge of R_S is necessary to compute $A_{\mathcal{G}}$. To estimate R_S, we analyze the eigen decomposition of the covariance matrix $R_{\mathcal{O}}$ as stated in equation (5).

$$\mathrm{R}_{\mathcal{O}} = \mathrm{E}_{\mathcal{O}}\Lambda\mathrm{E}_{\mathcal{O}}^{H} \qquad (10)$$

The matrix $\mathrm{E}_{\mathcal{O}}$ is defined as the concatenation of the eigenvector matrices$\mathrm{E}_{\mathcal{O}}$ and $\mathrm{E}_{\mathcal{O}n}$, which represent the signal and noise subspaces, respectively. $\mathrm{R}_{\mathcal{O}}$ is a matrix whose eigenvalues are arranged in Λ a diagonal matrix. The value is 5. may also be expressed as

$$\mathbf{R}_{\mathcal{O}} = \mathbf{E}_{\mathcal{O}_s}\mathbf{\Lambda}_s\mathbf{E}_{\mathcal{O}_s}^{H} + \mathbf{E}_{\mathcal{O}_n}\mathbf{\Lambda}_n\mathbf{E}_{\mathcal{O}_n}^{H} \qquad (11)$$

Where both $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$ diagonal matrixes$\in \mathbb{C}^{k\times k}$ and$\mathbb{C}^{(2M+N-1-k)\times(2M+N-1-k)}$ composed to $\mathbf{R}_{\mathcal{O}}$ with respect to signal and noise respectively. Using (5)and (8) $\mathbf{R}_{\mathcal{O}}$ can be rewrite as the following

$$\widehat{\mathbf{R}}_S = \widehat{\mathbf{A}}_{\mathcal{O}}^{\dagger}\mathbf{E}_{\mathcal{O}_s}\mathbf{\Lambda}_s\mathbf{E}_{\mathcal{O}_s}^{H}\left(\widehat{\mathbf{A}}_{\mathcal{O}}^{H}\right)^{\dagger} \qquad (12)$$

Where $(.)^{\dagger}$ represent Moore-Penrose pseudo-inverse operation. Finally , the steering matrix $A_{\mathcal{G}}$ is derived from (3.45) it closed form can be as the following

$$\widehat{\mathbf{A}}_{\mathcal{G}} = \left(\widehat{\mathbf{R}}_S^{-1}\left(\widehat{\mathbf{A}}_{\mathcal{O}}\right)^{\dagger}\mathbf{R}_{\mathcal{O}\mathcal{G}}\right)^{H} \quad (13)$$

Using (12) and (13) we can obtain

$$\widehat{\mathbf{A}}_{\mathcal{G}} = \left(\left(\widehat{\mathbf{A}}_{\mathcal{O}}^{\dagger}\mathbf{E}_{\mathcal{O}_s}\mathbf{\Lambda}_s\mathbf{E}_{\mathcal{O}_s}^{H}\left(\widehat{\mathbf{A}}_{\mathcal{O}}^{H}\right)^{\dagger}\right)^{-1}\widehat{\mathbf{A}}_{\mathcal{O}}^{\dagger}\mathbf{R}_{\mathcal{O}\mathcal{G}}\right)^{H} \qquad (14)$$

It should be noted that the size of the estimated steering matrix is equal to $(2M + N - 1) \times K$. In the scenario when the problem is under-determined, the size of the steering matrix is less than K, specifically (2M + N - 1)< K. In this particular scenario, the covariance matrix does not provide correct outcomes because of its rank-deficiency. Instead, we use the columns of $\widehat{\mathbf{A}}_{\mathcal{G}}$ to individually estimate the elevation angles, ensuring that each elevation angle is matched with its associated azimuth angle. Therefore, the MUSIC method may be used to estimate the elevation angles from $\widehat{\mathbf{R}}_{\mathcal{G}}$may be acquired for the kth column of $\widehat{\mathbf{A}}_{\mathcal{G}}$. Given that rank[$\widehat{\mathbf{R}}_{\mathcal{G}}$] = 1, the 1-D MUSIC method is used to estimate the value of θ_k. Specifically, the value of ϑ_k is calculated from

$$\vartheta_k = \arg\max_{\vartheta} \frac{1}{a_{\mathcal{G}}^{H}(\vartheta)G_kG_k^{H}a_{\mathcal{G}}(\vartheta)} \quad (15)$$

B. Deep Learning for V-Shape

In this model its use a deep Convolutional Neural Network (CNN) to undergo training in order to acquire the ability to accurately forecast the Direction of Arrival (DOAs). The convolution layers capture features from the multi-channel input data, while the fully connected (FC) layers use the output of the convolution layers to make Direction of Arrival (DOA) estimations using a pre-determined grid. The prediction of (DOA) is formulated as a challenge of multilabel classification. For each angle $\theta_k$in the set {1◦, . . . , 90◦}, we examine a grid $\mathbb{E}$ consisting of $2\mathbb{E}+1$ discrete points with a resolution of ρ degrees. This grid G is defined as {- $\mathbb{E}$ ρ, . . . , -ρ, 0◦, ρ, . . . , $\mathbb{E}$ ρ}, which falls within the range of [-90◦, 90◦]. Each point on the grid $\mathbb{E}$ corresponds to the angle θ_k. For every signal-to-noise ratio (SNR) level, K angles are chosen from the set $\mathbb{E}$, and the corresponding covariance matrix is computed using equation (3.41). The input data for the proposed Convolutional Neural Network (CNN) is a matrix of size N × N × 3, where each element is a real number. The third dimension of the matrix indicates distinct channels. The first and second channels represent the real and imaginary components $\boldsymbol{R}_{\mathcal{O}} and\ \boldsymbol{R}_{\mathcal{G}}$, denoted as $X_{:,:,1}$ = Re{$\boldsymbol{R}_{\mathcal{O}} and\ \boldsymbol{R}_{\mathcal{G}}$} and $X_{:,:,2}$ = Im{$\boldsymbol{R}_{\mathcal{O}} and\ \boldsymbol{R}_{\mathcal{G}}$}, respectively. The third channel corresponds to the phase entries, denoted as $X_{:,:,3}$ = $\angle${$\boldsymbol{R}_{\mathcal{O}} and\ \boldsymbol{R}_{\mathcal{G}}$}. The input data to the CNN consists of a collection of D data points, denoted as X = {$X_{(1)}$, . . . , $X_{(D)}$}. For each example $X_{(i)}$, the K training angles in E are converted into a binary vector where K elements are set to one and the remaining elements are set to zero. For instance, if the maximum angle $\theta_k$is 60 degrees and the required resolution is ρ = 1 degree, the grid will consist of 121 grid points ranging from -60 degrees to 60 degrees. In addition, the angle pair {−60◦, −59◦} corresponds to the 121 × 1 binary vector z = [1, 1, 0, . . . , 0]T, which acts as the appropriate label or output of the proposed CNN. Therefore, the i-th label $z_{(i)}$ is a member of the set Z = $\{0,1\}^{2E+12}$, as determined by the given approach. Therefore, each training example, denoted as the i-th example, is represented by pairs in the format ($X_{(i)}$, $z_{(i)}$), resulting in a training data set $\mathcal{D} = \{(\mathcal{X}_{(1)}, z_{(1)}), (\mathcal{X}_{(2)}, z_{(2)}), .., (\mathcal{X}_{(\mathcal{D})}, z_{(\mathcal{D})})\}$ with a total of $\mathcal{D}$ elements.

The well-established universal approximation theorem states that a feed-forward network, consisting of a single hidden layer and processed by a multilayer perceptron, has the capability to approximate continuous functions on a compact subset of $\mathbb{R}^n$. The objective of this multilabel classification problem is to generate a machine learning hypothesis, which is represented as a function that maps the input space to the output space. In other words, : $\mathbb{R}^{N\times N\times 3} \rightarrow \mathcal{Z}$. While the network is trained using the actual covariance matrix, during testing and evaluation, the sample covariance from equation (3.44) is utilized since the actual covariance is not known. During the testing phase of the CNN, all input samples may be regarded as "unseen data" in relation to the training.

In this model we use 24 layers for the CNN layer {1,4,7,10} use 2D CNN whom filter is 256 with kernel size is $\mathscr{k} \times \mathscr{k}$ the first layer its kernel is 3 while the rest of the layer is 2.Meanwhile the stride we use is $\delta \times \delta$ for the first layer is 1 and for the rest is 2. The convolution can be expressed as the following

$$(\mathrm{X} * \mathbf{K})_{m,n} = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{3} K_{i,j,k} X_{m+i-1,n+j-1,k} \qquad (16)$$

The design of the deep learning model is shown in figure 2

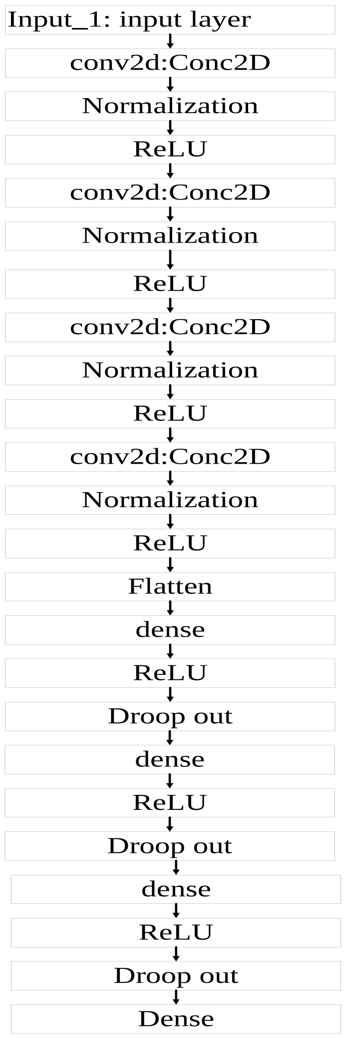

Figure2: The implemented deep network for 2D V-shape coprime array

III. Result

In this section will be demonstrated the result of 2D coprime array with array geometry of V-shape coprime array the result will be compared with other type of 2 D geometry .

For VCA it requires less sensor to solve same number of sources the number of sensors in each method will be presented in table below 1

Table 1 Number of source element can be resolved by arrays sensor

ULA	N-1
LCA	MN
CPA	Min (N^2, M^2)
VCA	MN

Also, in table 4-2 will demonstrate the real number of sensors

Table 4-2 number of sensors in array

ULA	N
LCA	2M-N-1
CPA	$N^2 + M^2$
VCA	4M-2N-3

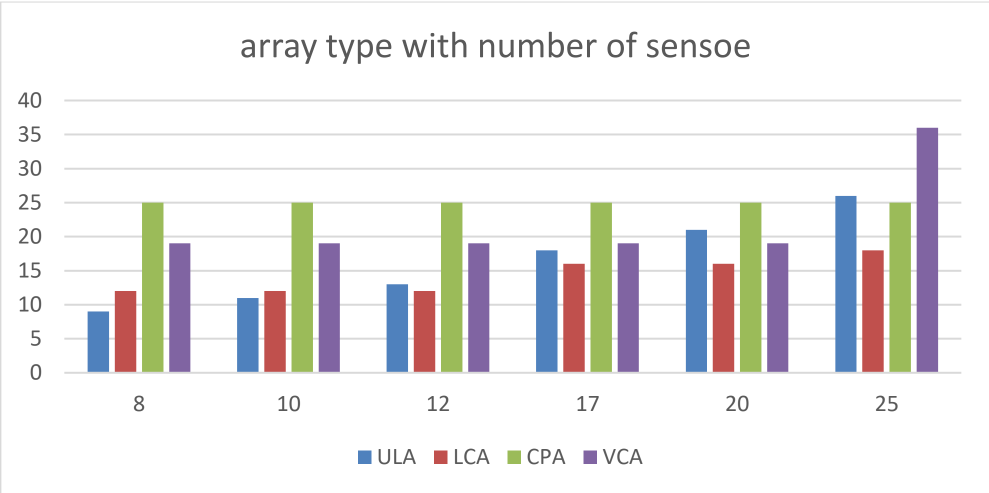

Figure 3:comparission between different geometry for the same number of antenna array

Spectrum for the suggested model is shown in figure 4

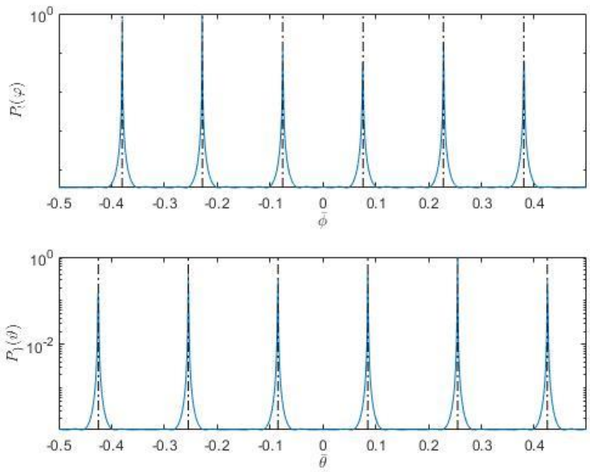

Figure 5: the spectrum of the suggested system

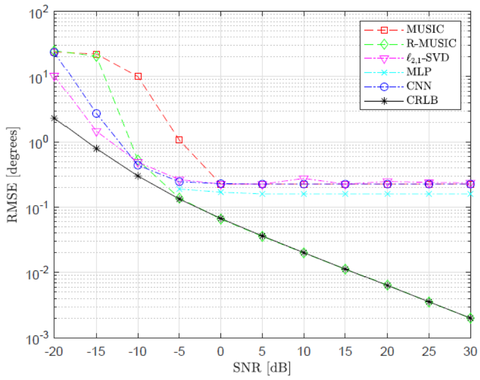

Figure 6:RMSE for the proposed system with other method with low SNR

The performance of the suggested CNN model for 1000 (MC) at each level of the SNR with 1000 snapshot The Root Mean Square Error (RMSE) findings are shown in Figure 6. Within the low (SNR) range, CNN has a well-balanced performance in relation to the (RMSE), which is comparable to that of the robust $\ell_{2,1}$ SVD method, except for the lowest SNR value at-20 dB. However, unlike the later procedure, no adjustment of any parameters is required. SNR is essential for CNN, providing a significant benefit in real-world applications. In the high (SNR) domain, the root mean square error (RMSE) reaches its minimum value for the grid-based approaches, while only the gridles methods do not reach this minimum. The R-MUSIC estimator is capable of achieving the Cramér-Rao lower bound (CRLB). This performance is applicable to all approaches that are based on a grid and is not subject to variation. can only be enhanced by using a more refined grid. It is important to acknowledge that the CNN successfully predicts Despite lacking training in such scenarios, the angle estimations are still adequate at high signal-to-noise ratios. In addition, we have graphed the MLP RMSE values for a signal-to-noise ratio (SNR) of -5. The DOAs could not be reliably resolved at lower SNRs, so the minimum required SNR for consistent resolution is dB and above

IV. Conclusion

The proposed scheme use of CNN to implement feature extraction from the covariance matrix of the received signal. Their training is over simulated coprime array signals, and thus the method generalizes well. Simulation results show that the proposed method improves performance remarkably compared to conventional methods of DOA such as MUSIC and root MUSIC. The proposed method is noise- and clutter-insensitive.

References

[1] Y. M. Tabra and B. M. Sabbar, "Hybrid MVDR-LMS beamforming for massive MIMO," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 16, no. 2, pp. 715–723, 2019, doi: 10.11591/ijeecs.v16.i2.pp715-723.

[2] F. Salman and B. M. Sabbar, "Low-Complexity DOA Estimation Method Based on Joined Coprime Array," *Journal of Telecommunications and Information Technology*, 2024, [Online]. Available: https://api.semanticscholar.org/CorpusID:267186479

[3] P. P. Vaidyanathan and P. Pal, "Sparse Sensing With Co-Prime Samplers and Arrays," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 573–586, 2011, doi: 10.1109/TSP.2010.2089682.

[4] A. Moffet, "Minimum-redundancy linear arrays," *IEEE Trans Antennas Propag*, vol. 16, no. 2, pp. 172–175, 1968, doi: 10.1109/TAP.1968.1139138.

[5] P. Pal and P. P. Vaidyanathan, "Coprime sampling and the music algorithm," in *2011 Digital Signal Processing and Signal Processing Education Meeting (DSP/SPE)*, 2011, pp. 289–294. doi: 10.1109/DSP-SPE.2011.5739227.

[6] J. Zhao and C. Ritz, "Coprime Microphone Arrays for Estimating Speech Direction of Arrival Using Deep Learning," in *2021 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*, 2021, pp. 1–8.

[7] A. M. Elbir, “V-Shaped Sparse Arrays For 2-D DOA Estimation,” Dec. 2019, doi: 10.1007/s00034-018-0991-5.

[8] A. M. Elbir, “DeepMUSIC: Multiple Signal Classification via Deep Learning,” Dec. 2019, doi: 10.1109/LSENS.2020.2980384.

[9] I. Goodfellow, Y. Bengio, and A. Courville, “Deep Learning.”

[10] S. Ge, K. Li, and S. N. B. M. Rum, “Deep Learning Approach in DOA Estimation: A Systematic Literature Review,” *Mobile Information Systems*, vol. 2021, 2021, doi: 10.1155/2021/6392875.

[11] S. Zheng *et al.*, “Deep Learning-Based DOA Estimation,” *IEEE Trans Cogn Commun Netw*, vol. 10, no. 3, pp. 819–835, 2024, doi: 10.1109/TCCN.2024.3360527.

[12] J. Yu and Y. Wang, “Deep Learning-Based Multipath DoAs Estimation Method for mmWave Massive MIMO Systems in Low SNR,” *IEEE Trans Veh Technol*, vol. 72, no. 6, pp. 7480–7490, 2023, doi: 10.1109/TVT.2023.3239402Article in a conference proceedings: