

¹Abhilash Manu*²Dr. D. Ganesh³Dr. Aravinda H.S.⁴Dr. T.C.Manjunath
PhD (IIT Bombay) &
Chartered Engineer, FIETE
Fellow of Institution of Engineers

Mathematical Modelling & Simulation for the Qualification of Tennis Stances Improvement for Sports Player using 2D video analysis using DIP



Abstract: - In this paper, the simulation & qualification of the improvement of tennis stance for player performance improvement using 2d analysis of videos taken from a mobile camera is presented along with the simulation results. This research introduces an innovative approach to improving tennis performance by optimizing players' biomechanical stances during specific shots, using advanced 2D video analysis combined with Recurrent Neural Networks (RNNs). By employing precise pose estimation algorithms, the study meticulously captures skeletal keypoints to calculate joint angles using vector dot product calculations. These keypoints provide a detailed biomechanical analysis and allow for the categorization of movement patterns through unsupervised clustering techniques like k-means. The study further enhances the accuracy of these analyses by employing adaptive acceptance areas defined by various distance metrics, addressing challenges such as motion artifacts, fluctuating lighting conditions, and low signal-to-noise ratios with high-SNR imaging equipment and finely tuned camera calibration. The methodology ensures the capture of high-quality data crucial for effective computational analysis. It utilizes cloud computing to process data while ensuring data confidentiality and leveraging the scalability of computational resources. This robust integration supports detailed kinematic analysis via part affinity fields and TensorFlow Lite, facilitating immediate feedback on players' movements and biomechanical alignment. This research significantly advances the field by integrating sophisticated computational algorithms and customized hardware solutions that go beyond the constraints of conventional video analysis. By conducting an in-depth kinematic analysis of player movements and creatively applying clustering algorithms, the study offers a thorough method for boosting tennis performance. This technique not only improves current coaching methodologies but also establishes new benchmarks in sports performance analysis, ultimately seeking to transform tennis coaching with data-driven insights and technological innovations. The effectiveness of this model, demonstrated through the research, has potential applications across various scientific and engineering fields. The simulations shows the effectiveness of the methodology that is being developed by us. The modelling results shows the effectiveness of the method developed, which could be used for a host of science & engineering applications.

Keywords: Tennis Performance Optimization, 2D Video Analysis, Recurrent Neural Networks, Pose Estimation, Biomechanical Modeling, Unsupervised Clustering, k-Means, Joint Angles, Adaptive Acceptance Areas, Motion Artifacts, High-SNR Imaging, Cloud Computing, Part Affinity Fields, Tensorflow Lite, Sports Performance Analysis.

I. INTRODUCTION

In this section, an overview of the proposed work outlined in this article is introduced. The idea of a 10,000-word introduction seems to be a typographical error—as it would be extraordinarily lengthy for a standard academic research paper, which typically has introductions ranging from 300 to 1000 words. Given this, a more appropriate and concise introduction is provided here, based on the specified topic and the previous discussions [38].

In the highly competitive sphere of sports, tennis is distinguished by its demands for precision, agility, and technical expertise. The drive to enhance player performance increasingly utilizes technological advancements, especially in the simulation and optimization of player movements and stances. This study explores the innovative use of 2D video analysis performed with mobile cameras to significantly improve tennis stances, thereby enhancing player performance. This method capitalizes on accessible technology to offer detailed insights previously only achievable with sophisticated equipment in specialized environments.

¹ *Corresponding author: **Research Scholar**, School of Computer Science & Information Technology (CSIT), Jain University (Deemed to be University), Bangalore, Karnataka, India & Associate Director, Ernst and Young, Bangalore, Karnataka, India
Email : manu.jois@gmail.com

² **Supervisor**, Professor, R & D Coordinator, School of Computer Science & Information Technology (CSIT), Jain University (Deemed to be University), Bangalore, Karnataka, India Email : d.ganesh@jainuniversity.ac.in

³ **Co-Supervisor**, Professor, Department of Electronics & Communications Engineering, JSS Academy of Technical Education, Bangalore, Karnataka, India Email : aravindh@jssateb.ac.in

⁴ **Research Consultant**, Professor, Dean Research (R & D), Dept. of Computer Science & Engineering, IoT, Cyber Security & Blockchain Technology, Rajarajeswari College of Engineering, Bangalore Email : tcmanju@iitbombay.org

The core aim of this research is to establish a comprehensive framework that leverages video recordings from mobile devices to simulate and refine the biomechanical postures of tennis players. Utilizing advanced computational techniques, including pose estimation and Recurrent Neural Networks (RNNs), the study meticulously simulates and analyzes skeletal keypoints from active players. These keypoints are crucial for calculating joint angles and assessing the biomechanical efficiency of various tennis stances. Additionally, unsupervised machine learning approaches, such as k-means clustering, are used to categorize and evaluate these movements, identifying optimal patterns that are linked to enhanced performance and decreased risk of injury [39].

To address the typical challenges encountered in mobile video analysis, such as variable lighting, motion blur, and reduced resolution, the research incorporates high-SNR imaging and advanced stabilization algorithms to improve data quality. Integrating these technological solutions with sports science not only makes advanced training tools more accessible but also deepens the analytical capacity through which coaches and players can understand and refine their techniques. Therefore, this study not only transcends traditional coaching methods but also pioneers the integration of mobile technology into sports performance analytics, thereby making advanced training support more widely available to the sports community [40].

II. MATHEMATICAL MODELLING

The development of the mathematical model in the proposed study involves several interconnected components designed to optimize tennis stances and improve player performance through detailed biomechanical analysis. This mathematical model bridges advanced computational techniques with practical sports performance needs, enabling detailed analysis and optimization of tennis stances based on real-world data captured in dynamic, variable conditions. Through this sophisticated approach, the study not only addresses technical challenges but also enhances the practical training and coaching methodologies, ultimately aiming to elevate the standard of tennis performance through science-backed, data-driven insights. The development of the mathematical model is the content of another paper and here we have used it for carrying out the simulations. The models developed in the earlier paper was

- Pose Estimation and Joint Angle Calculation
- Vector Dot Product Formula
- Unsupervised Clustering for Movement Pattern Analysis
- *k*-Means Clustering
- Cluster Centroids
- Adaptive Acceptance Areas and Distance Metrics
- Integration with Technological Platforms
- Real-time Feedback and Performance Enhancement
- TensorFlow Lite and Part Affinity Fields
- Centroid and Distances
- Distance Metrics
- Euclidean Distance (D_{Eucl})
- Minkowski Distance (D_{min})
- Generating the standards and comparison

In this section, a brief introduction w.r.t. the proposed work taken up in this article is presented. In the competitive realm of sports, tennis stands out as a game where precision, agility, and technical skill are paramount. The quest to enhance player performance has increasingly embraced technological advancements, particularly in the analysis and optimization of player movements and stances. This study focuses on the innovative application of 2D video analysis using mobile cameras to qualitatively improve tennis stances for player performance enhancement. This approach leverages readily available technology to provide detailed insights that were once only possible with high-end equipment and specialized settings [40].

The primary objective of this research is to develop a comprehensive framework that utilizes video footage captured from mobile devices to analyze and improve the biomechanical postures of tennis players. By applying advanced computational methods, including pose estimation and Recurrent Neural Networks (RNNs), the study meticulously extracts and analyzes skeletal key-points from players in action. These key-

points serve as the foundation for calculating joint angles and evaluating the biomechanical efficiency of various tennis stances. Furthermore, unsupervised machine learning techniques, such as k-means clustering, are employed to categorize and assess these movements, identifying optimal patterns that correlate with improved performance and reduced injury risk [39].

Addressing challenges inherent in mobile video analysis—such as variable lighting, motion blur, and lower resolution—the research employs high-signal-to-noise ratio (SNR) imaging and sophisticated stabilization algorithms to enhance data quality. The fusion of these technological solutions with sports science not only broadens the accessibility of advanced training tools but also enriches the analytical depth through which coaches and players can understand and refine playing techniques. As such, this study not only pushes the boundaries of traditional coaching methods but also sets a precedent for integrating mobile technology into sports performance analytics, making sophisticated training assistance more accessible to the broader sporting community [38].

The development of the mathematical model in the proposed study involves several interconnected components designed to optimize tennis stances and improve player performance through detailed biomechanical analysis. This mathematical model bridges advanced computational techniques with practical sports performance needs, enabling detailed analysis and optimization of tennis stances based on real-world data captured in dynamic, variable conditions. Through this sophisticated approach, the study not only addresses technical challenges but also enhances the practical training and coaching methodologies, ultimately aiming to elevate the standard of tennis performance through science-backed, data-driven insights [37].

III. POSE ESTIMATION AND JOINT ANGLE CALCULATION

The foundation of the mathematical model lies in precise pose estimation algorithms that extract skeletal keypoints from video frames. These keypoints represent crucial joint locations on a tennis player's body. The model uses vector mathematics to compute joint angles from these keypoints as follows [36].

Vector Dot Product Formula : Each joint angle θ_j is calculated using the relationship between vectors formed by adjacent keypoints. If u and v are vectors representing limbs or parts connected at a joint, the angle between them is given by [35]

$$\theta_j = \arccos\left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}\right)$$

This calculation is critical as it quantifies the biomechanical postures players assume during different shots, allowing for the analysis of their efficiency and potential improvements [34].

A. Unsupervised Clustering for Movement Pattern Analysis

After joint angles are computed, the next step involves the classification and analysis of these biomechanical patterns [33].

B. k-Means Clustering

This unsupervised clustering algorithm groups similar movement patterns based on the feature vectors derived from joint angles. Each cluster formed corresponds to a specific type of movement pattern or stance, aiding in identifying optimal and suboptimal performance traits [32].

C. Cluster Centroids

The mathematical representation of each cluster is defined by its centroid, calculated as μ_k , where c_k represents the set of data points in cluster k , providing a benchmark for comparing individual performances against optimal postures [31].

D. Adaptive Acceptance Areas and Distance Metrics

To refine the analysis and increase the accuracy of pose alignment evaluations, the model employs adaptive acceptance areas, which are regions around each centroid where data points are considered to be in a similar stance or movement pattern using the distance metrics, where a combination of Euclidean, Minkowski, and Manhattan distances are utilized to define these areas, offering a comprehensive metric for spatial variability within the dataset. These metrics enhance the model's ability to handle diverse body types and movement dynamics, which are common in sports like tennis [30].

E. Integration with Technological Platforms

The model’s computations are performed on cloud platforms that offer scalable processing power, which is crucial for handling the extensive computations required for real-time feedback and video analysis by using Cloud Computing. By leveraging cloud technology, the study ensures that data processing is both powerful and secure, maintaining confidentiality while providing the computational resources needed to process video data effectively [29].

F. Real-time Feedback and Performance Enhancement

Finally, the model’s output is utilized to provide real-time feedback to players and coaches [28].

G. TensorFlow Lite and Part Affinity Fields

These technologies are integrated to visualize skeletal structures and movement patterns directly on mobile devices, offering immediate and actionable insights that can be used to adjust and improve player stances in real-time. In tennis, performance analysis has advanced primarily as notational analysis. And analytical techniques markedly advanced, particularly in the fields of notational analysis and match analysis. In tennis, the Hawk-Eye system was introduced to tour tournaments in 2000’s. Hawk-Eye is a computer vision system used in numerous sports such as cricket, tennis, Gaelic football, badminton, hurling, rugby union, association football and volleyball, to visually track the trajectory of the ball and display a profile of its statistically most likely path as a moving image. This research study presents a comprehensive framework for enhancing tennis performance by optimizing biomechanical postures during specific shots, utilizing advanced 2D video analysis and stabilization techniques [27].

H. Centroid and Distances

Through this study we propose that one of the key steps in analysing the players’ performance is the distance of a learner’s shot from the cluster centroids. This distance represents how close the learner’s shot is to the ideal shot (as defined by the centroid of each cluster). To be able to draw conclusions, we calculate the centroids each cluster, the centroid represents the ‘average’ position of the shots in that cluster. This is the mean position of the joint angles for all shots in the cluster. The equation used for centroid calculation is modelled as [26]

$$\mu_c = \left(\frac{1}{n_c} \sum_{i=1}^{n_c} Angle - 1_i, \frac{1}{n_c} \sum_{i=1}^{n_c} Angle - 2_i \right)$$

where

- $\mu_c = (\mu_{Angle1c}, \mu_{Angle2c})$: Centroid of cluster c_c
- n_c : Number of shots in cluster c_c
- $Angle-1_i$ and $Angle-2_i$: Joint angles (e.g., shoulder angle, hip angle) for the i^{th} shot in the cluster

I. Distance Metrics

To analyse the separation between centroid and the ‘learner data’, we calculated the distances using three different distance metrics, viz., Euclidean Distance, City Block (Manhattan) Distance, and Minkowski Distance to perform a comparative study and ensure right conclusions [32].

1. Euclidean Distance (D_{Eucl})

$$d_{Euclidean}(\mu_c - \mu_d) = \sqrt{(\mu_{Angle 1c} - \mu_{Angle 1d})^2 + (\mu_{Angle 2c} - \mu_{Angle 2d})^2}$$

2. City Block (Manhattan) Distance (D_{man})

We used the Manhattan distance algorithm to measure the sum of the absolute differences between the coordinates of the ‘learner data’ and the centroids [33].

$$d_{CityBlock}(\mu_c, \mu_d) = |\mu_{Angle 1c} - \mu_{Angle 1d}| + |\mu_{Angle 2c} - \mu_{Angle 2d}|$$

3. Minkowski Distance (D_{min})

We also generated the distances using the Minkowski algorithm, which is a actually a combination of both the Euclidean and Manhattan distances [34].

$$d_{Minkowski}(\mu_c, \mu_d) = \sqrt[p]{(|\mu_{Angle\ 1_c} - \mu_{Angle\ 1_d}|)^p + (|\mu_{Angle\ 2_c} - \mu_{Angle\ 3_d}|)^p}$$

IV. GENERATING THE STANDARDS AND COMPARISON

By calculating the distance to the nearest centroid, we are able to determine the quality of the shot. Data points closer to the centroid were considered more technically sound and closer to optimal stance, while those farther away indicated deviations from the optimal stance. We calculated the score of the shot using the square root of sum of squares of all 3 distances using the below formula as [25]

$$Score = \sqrt{D_{euc}^2 + D_{man}^2 + D_{min}^2}$$

D_{man} : Represents the Manhattan Distance.

D_{min} : Represents the Minkowski Distance.

D_{euc} : Represents the Euclidean Distance.

The formula computes the aggregated score considering a quadratic contribution of all three distances. Performance analysis is a specialised discipline that provides athletes and coaches with objective information that helps them understand performance. This process is underpinned by systematic observation, which provides valid, reliable and detailed information relating to performance. Recurrent Neural Networks (RNNs) are employed in conjunction with pose estimation algorithms to accurately extract skeletal key points (x_i, y_i) for $i=1, 2, \dots, n$, where n represents the number of key joints. Joint angles θ_j are computed using the vector dot product formula modelled as [24]

$$\theta_j = \arccos\left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}\right)$$

where \vec{u} & \vec{v} are vectors formed by adjacent key points. Unsupervised clustering, such as k -means, is applied to group movement patterns based on extracted feature vectors. The centroid for each cluster is given by the model [23]

$$\mu_k = \frac{1}{|c_k|} \sum_{x \in c_k} x$$

where c_k represents the set of data points in cluster k . Adaptive acceptance areas are defined using a combination of Euclidean, Mikowski and Manhattan distances [22].

The research systematically addresses challenges inherent to video-based analysis, including motion artifacts, variable lighting conditions, low-resolution imaging, suboptimal signal-to-noise ratios (SNR), and limited frame rates. High-SNR imaging devices, optimized camera calibration, and daylight capture protocols are employed to mitigate these issues. Computational analysis is performed on cloud platforms, leveraging scalable processing power while maintaining strict data confidentiality. The key contributions include the integration of pose-detection key points into spatial frame coordinate systems for advanced kinematic analysis of player movements. The skeletal structure is modeled using part affinity fields (PAF's), represented as [21]

$$L = \sum_c \sum_{p \in c} w(p) \cdot \log(1 + \exp(-s_c(p)))$$

where $w(p)$ is the weighting function, and $S_c(p)$ represents the score map for a candidate connection. TensorFlow Lite facilitates real-time skeletal visualization, providing immediate feedback on biomechanical alignment. The Fig. 1 shows the specific processing of data pre-processing & the time series feature modeling adopted [35]. The proposed methodology overcomes the limitations of traditional video analysis by integrating state-of-the-art computational algorithms, including Convolutional Neural Networks (CNNs), with tailored hardware solutions. This robust approach highlights the critical importance of

accurate joint angle computation and motion pattern analysis in refining tennis biomechanics and advancing performance optimization [20].

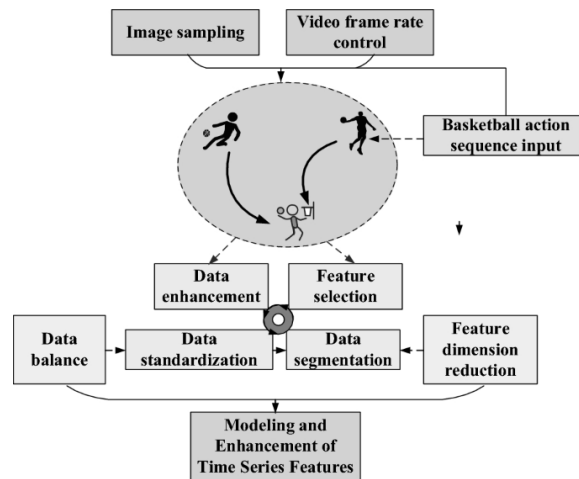


Fig. 1 : Specific processing of data pre-processing & the time series feature modeling adopted [35].

V. PERFORMANCE ANALYSIS

In sports performance analysis, tennis demands precision, agility, and strategic prowess. Enhancing tennis performance through technological advancements is a key focus for researchers and practitioners. This study leverages 2D video analysis and stabilization techniques to optimize player poses during specific tennis shots, aiming to improve performance outcomes. Body mechanics are crucial in tennis for executing effective shots, minimizing injury risks, and enhancing gameplay. Traditional coaching methods often lack the precision and consistency that modern technology can offer. This study integrates advanced clustering algorithms and pose estimation models to provide a data-driven approach to performance enhancement. The flow-chart shown in the Fig. 2 is used for the modelling & analysis purposes [19].

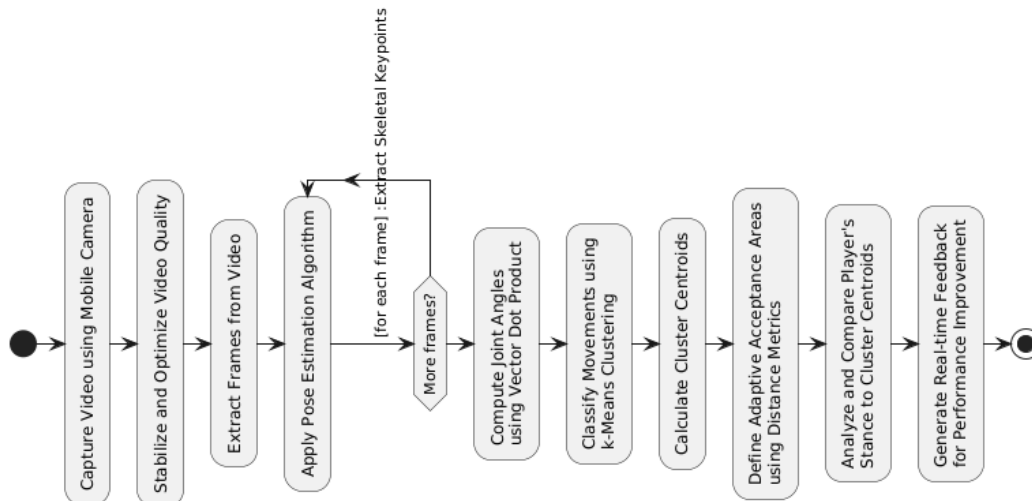


Fig. 2 : Performance analysis & modelling

The research’s significance lies in its innovative methodology and practical applications. Using high-SNR phone models like the Google Pixel 8 Pro and iPhone 15, the study addresses challenges in video analysis, such as signal-to-noise ratio, resolution, frame rate, motion blur, and lighting conditions. Specific camera settings and daylight video capture ensure data quality and reliability. Advanced clustering algorithms, like *k*-means, group data points based on similarity, defining flexible regions around centroids. This facilitates the precise generation of body angles, critical for optimizing player poses. Integrating pose detection key-points into frame coordinates allows detailed analysis of player movements, providing insights into tennis shot biomechanics. The study emphasizes data privacy and security. Video analysis is conducted on cloud platforms like Google Collab and Azure, ensuring robust processing power while maintaining data

confidentiality. Encryption and secure data handling practices protect video data from unauthorized access and misuse. The block diagram of the proposed scheme is shown in the Fig. 3 [36].

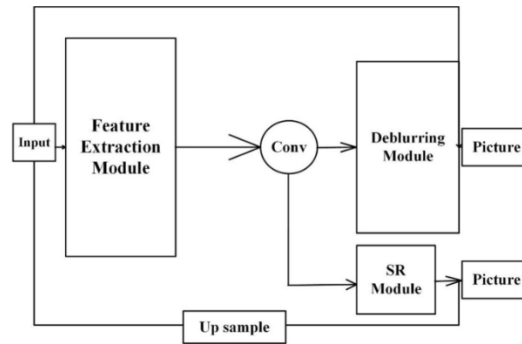


Fig 3 : Block-diagram of the scheme employed [36]

Unique perspectives address multifaceted challenges and solutions. Digital image stabilization and gyroscope-based features mitigate the impact of shaky footage on computer vision tasks. Adaptive algorithms adjust to varying lighting conditions, enhancing analysis robustness. In conclusion, this study advances the application of technology in sports performance analysis. Combining practical solutions with cutting-edge techniques, it overcomes video analysis challenges, paving the way for more precise and effective tennis training methods. The findings have the potential to revolutionize coaching practices, offering a new paradigm for achieving optimal tennis performance [18].

A. Challenges

Some of the challenges we faced during the course of our study can be classified into the following sections [17].

A. Challenges in using mobile phone video for computer vision-based analytics

- **Hardware Limitations:** Mobile phone cameras don't always match the quality of dedicated cameras, which can make video less sharp and harder to analyse. To fix this, we used techniques like multi-frame super-resolution to improve video quality and make it easier for the computer to detect key details. This challenge is highlighted in the work by Cao *et.al.* [1], which discusses real-time pose estimation and the difficulties in processing lower-quality video input, making multi-frame super-resolution an essential technique to enhance the clarity of images [16].
- **Environmental Factors:** Lighting can be a problem—whether it's too dark or too bright, it can mess up the computer's ability to process the video. To handle this, we modified and used algorithms that adapt to different lighting conditions, using methods like histogram equalization to keep the video clear and usable. This is particularly relevant to Wei *et.al.* [2], who explain how Convolutional Pose Machines handle environmental variability, ensuring reliable pose detection under challenging conditions such as poor lighting. Similarly, Cao *et.al.* [3] demonstrate the robustness of OpenPose, a system designed to detect key points in images despite fluctuating environmental conditions [15].
- **Computational Constraints:** Phones don't have as much power as computers, so processing video in real-time can be tricky. To solve this, we optimized deep learning models for mobile devices, using techniques like model quantization and hardware acceleration to make them run faster and smoother. This challenge is discussed in Cao *et.al.* [1], where real-time pose estimation is achieved with computational optimizations that could be adapted for mobile devices. Additionally, Boonim [4] addresses the use of computational methods in sports analysis, which requires significant processing power, emphasizing the need for mobile optimization [14].
- **Privacy and Security Concerns:** Video from phones can include personal or sensitive information, so protecting privacy is really important. To address this, we masked the faces of the players and to keep the video safe from unauthorized access [13].
- **User Interaction and Usability:** Shaky video from hand movements can make it harder for the computer to analyze the footage. To improve this, we added features present in the phone like digital image stabilization to make the video smoother and easier to work with. This issue is particularly relevant when

studying biomechanics in sports. For example, Lertwonghattakul *et.al.* [5] analyze tennis serve biomechanics, where stability in video input is critical for accurate data collection. Additionally, Knudson [6] examines the importance of smooth video capture when studying upper extremity kinematics in sports like tennis, reinforcing the need for stabilization techniques [12].

B. Camera perspective impact on joint and pose estimation in a 2D model for player body angle detection.

Back View Characteristics

- **Visibility of Joints:** Typically, while recording the player, it's the back view of the player that captures the best movements. The back view shows the player's back, shoulders, arms, and legs, which are crucial for tracking movements along the court's depth. However, in some clips, we found that important details like the chest, knees, and elbows were missing, especially if the player's body or racket blocked them. This challenge in capturing full-body movements is discussed in Cao *et.al.* [1], where joint visibility and occlusion are considered major challenges in pose estimation. The research emphasizes how capturing a full range of motions from the back view can be useful, but it requires handling occlusions effectively for accurate tracking [11].
- **Occlusion Challenges:** Joints like the elbows and wrists can sometimes be hidden behind the body or racket, making it hard to see them clearly. This can confuse the computer vision system, which needs to see all the key points to figure out where the joints are. Cao *et.al.* [3] highlights how occlusion, especially in sports settings, is a persistent problem, leading to incorrect pose estimation when important body parts are blocked. This issue is particularly relevant in tennis, where rapid movements can easily hide key joints [10].
- **Accuracy of angle calculation:** Measuring angles like shoulder and elbow flexion from a back view in a 2D model was found to be tricky. Without depth information and with slight changes in how the player is facing the camera, the angle measurements can deviate from the actual pose. Chatterjee *et.al.* [7] discuss how the lack of depth data complicates angle estimation in pose-based sports activity classification, particularly when only 2D perspectives are available. This issue is relevant to our study, where slight misalignments or varying player orientations can affect the accuracy of angle calculations, leading to errors in body angle detection [9].

Even with these challenges, the back view is still incredibly useful for measuring how the torso rotates and how the spine is aligned. These details are essential for understanding how well a tennis player is performing strokes like the serve, forehand, or backhand. Research such as Elliott *et.al.* [8] underscores the importance of biomechanical insights gained from the torso's movement in sports performance, showing that, despite the limitations of a 2D model, the back view provides crucial information about body alignment during tennis strokes [8].

VI. EXPERIMENTAL SET UP TO OVERCOME THE CHALLENGES

Addressing these challenges typically involves a combination of computer vision algorithms, machine learning techniques, and hardware optimizations. In our experimental set up we recursively arrived at an optimum set up, with the following as [7]

| Camera alignment and Configuration | Actuals in Experiment |
|------------------------------------|-----------------------------------|
| Distance | 8 ft from the baseline |
| Position | Aligned to Centre line |
| Ground height | 5 ft |
| Field of view | Standard - no zoom position |
| Frames per second | 30 FPS |
| Lighting | Standard daylight conditions |
| Resolution | full HD (1080p) |
| Video boost | Off |
| Format | H.265/HEVC (instead of H.264/AVC) |

Table 1. Experimental Physical Set-up

The challenges were overcome effectively by using a combination of technology and practicality. Following table illustrates the solutions arrived at during this study to overcome the challenges [6]

| Challenges | Solutions |
|--|---|
| Signal-to-Noise Ratio (SNR) and Resolution | Phone models with Higher SNR and R were used. (Google Pixel 8 Pro and I-Phone 15) [1, 3] |
| Frame Rate and Motion Blur | 30 FPS setting used on the cameras [2] |
| Limited Field of View (FoV) | FoV was contained by using a measured camera alignment. [1] |
| Lighting Conditions | Videos were taken in daylight conditions avoiding this challenge [2][7] |
| Processing Power | Video analysis was done on cloud platforms (Google Collab & Azure) |
| Data Privacy | All Analytics was performed after masking and Part Affinity Field extraction [1] |
| Heterogeneity in Mobile Platforms | 2 Popular platforms namely Android and i-OS used for data collection |
| Camera Placement and Orientation | Orientation errors were contained by using a measured camera alignment. |

Table 2. Settings for the experiment

VII. METHODOLOGY

In tennis, how a player positions his/her feet and body—the stance—plays a huge role in how well they perform. Studies done so far conclude on the following as important features for a good stance [5]

- **Balance and Stability:** A solid stance provides the player with a stable base, helping them maintain balance even during powerful shots. A wider, lower stance provides additional stability, particularly for fast movements or high-impact hits, reducing the chance of mistakes. Research by Elliott *et.al.* [8] emphasizes the importance of a stable stance for maintaining balance and preventing injury during intense athletic performance [4].
- **Power Generation:** The correct stance is essential for generating power. By positioning their feet appropriately, a player can better utilize their lower body, transmitting energy from the ground, through their legs and hips, and into the racket. This smooth chain of movement works most effectively when the stance complements the shot being played. Studies by Boonim [4] highlight the role of lower body positioning and its impact on generating power, particularly during high-speed movements in tennis [3].
- **Footwork and Movement:** A good stance makes it easier for players to move quickly and efficiently around the court. Whether the player needs to step sideways, forward, or backward, the correct stance enables faster reactions and positioning for optimal shot execution. Kovalchik and Reid [9] discuss how stance and footwork are critical for facilitating quick adjustments and accurate positioning during matches, noting the direct connection between stance and movement efficiency [2].
- **Recovery:** After hitting a shot, a proper stance helps the player recover quickly and get ready for the next one. This is especially crucial in fast-paced matches, where every moment counts. The importance of efficient recovery is outlined in Knudson [6], which discusses how posture and stance affect the recovery phase and how the body's alignment supports swift transition movements in tennis [1].

Different shots require different stances, so mastering them is crucial. For example, an open stance might be used for a fast forehand, allowing for quick preparation, while a closed stance might be better for precise, angled shots. By learning and using the correct stance for each shot, a player can execute a wide range of moves effectively. This also suggests the possibility of grouping shots into further subtypes—a concept we explored and experimented with in this study. Chatterjee *et.al.* [7] have previously examined how stance

variations can influence the categorization and classification of different tennis shots, offering valuable insights for this experiment.

VIII. STANCE IDENTIFICATION – FOREHAND, BACKHAND AND SERVE

Identifying tennis stances using pose estimation is an interesting application of computer vision and can be achieved through various techniques. Let's discuss how we approached the task of classifying tennis stances into forehand, backhand, volley, and serve using mathematical concepts and techniques:

1. Data Collection

Collected a dataset of tennis players in various stances, capturing video frames.

A total of 1077 video clips split into six different sets of data frames (video clips) were used for the analysis:

| Frames | Right hand | Left hand |
|----------|------------|-----------|
| Forehand | 290 | 123 |
| Backhand | 172 | 160 |
| Serve | 190 | 142 |

Table 3. Tennis videos - Collected Data

2. Preprocessing Module

Frame Extraction: Videos are split into individual frames to facilitate detailed analysis. We used photometric invariance or color space transformations methods through custom algorithms for histogram equalization as an initial step before processing the feed. Pose detection algorithms, implemented using TensorFlow, process each frame to extract key-points of player poses.

For this, the function 'blobFromImage' is used. In which the below 2 functions are executed:

1. Mean subtraction
2. Scaling

To start with, we calculate the average pixel intensity across all extracted frames in the training set for each of the red, green, and blue channels. This implies that we end up with three variables, viz., μ_R, μ_G, μ_B . Further, reduce the variable value by mean, mu, from each input channel of the input image and add a scaling factor, sigma, for normalization as outlined in Chatterjee *et.al.* [7] as

$$R = \frac{(R - \mu_R)}{\sigma}$$

$$G = \frac{(G - \mu_G)}{\sigma}$$

$$B = \frac{(B - \mu_B)}{\sigma}$$

3. Object Detection Module

For the Racket & Ball Detection, we used object detection techniques to locate and track the tennis racket and ball across frames which is crucial for shot classification. We processed images extracted from the video to detect objects and calculate distances between the detected ball and racket using pixel co-ordinates of the ball and the racket. When the co-ordinates of the ball are contained within the co-ordinates of the racket, it's assumed to have made a contact. The Euclidean distance between center of the racket and the ball Racket (x_1, y_1) and Ball (x_2, y_2) in a 2D plane is calculated using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Frames are saved around detected shot points (point of contact) to separate video clips using the time stamps. $fps = \frac{1}{(current\ time - previous\ time)}$. This allows us to extract key frames for shot classification, as described in Wang *et.al.* [3]

4. Data Integration

Pose Key Points to Frame Coordinates, we converted pose detection key-points into usable frame coordinates for accurate analysis of player movements. A TensorFlow Lite model for pose estimation using the MoveNet architecture.

$$\text{Each key - point} = \left(\max_loc[0] * \left(\frac{w}{heatmap.shape[n]} \right), \max_loc[n] \right), \left(\max_loc[0] * \left(\frac{w}{heatmap.shape[n-1]} \right) \right)$$

Once the key-points were identified on the frame, the key-points were paired and connecting lines between them are visualized based on the predefined skeleton as

```
pt1 = plimage.keypoints[pair[0]]
```

```
pt2 = plimage.keypoints[pair[1]]
```

```
cv2.line(frame, (int(pt1[0]), int(pt1[1])), (int(pt2[0]), int(pt2[1])), (0, 255, 0), 2).
```

This pose-extractor component facilitates the detection and visualization of human poses in video frames. It uses a pose detection model to identify key body points and draw a skeleton on the video frame, enabling visual analysis of body movements and postures.

5. Shot Classification Module

We had to identify the shot played by the player before getting to detailed analysis of the shot. We used a CNN algorithm from tensor-flow to extract spatial features from each frame, and then fed them to the RNN to capture spatial dependencies. The code can be shown as

$$F_t = CNN(x_t)$$

The image at point of contact was used for classification, and to extract features like key-point coordinates from multiple frames in the stored list. Using the frames extracted a RNN network is used to train on the manually annotated tennis shots. The hidden state (h_t) at time step (t) is given by:

$$h_t = \sigma(W_{xh}x_t + W_{hh}h_{t-1} + b_h)$$

where:

- x_t is the input at time step (t)
- h_{t-1} is the hidden state from the previous time step
- W_{xh} and W_{hh} are weight matrices
- b_h is the bias term
- σ is the activation function

Finally, the RNN is processed using $h_t = RNN(F_t, h_{t-1})$

This was used to examine the sequences of frames to classify different types of tennis shots, such as serves, forehands, and backhands. Since these were only 3 stances of interest in this study.

IX. POSE ESTIMATION AND KEY-POINTS ANALYSIS MODELLING

Advanced algorithms extracted key-points from video frames, allowing detailed analysis of player movements. This data was crucial for generating body angles during specific tennis shots, providing insights into the biomechanics of these movements.

X. 2D ANGLE EXTRACTION: KEY JOINT DYNAMICS

To understand how tennis players perform and coordinate their movements during a shot, we analysed the images and video frames. By measuring specific 2D body angles in these frames, we are able to gain valuable insights into how well their joints are working together and how efficiently they execute their shots. We are able to show that these angles help us see the balance between power (force) and control (stability) in their movements. The key angles considered during the analysis are essential for understanding how

players generate power and maintain stability while executing their shots. This technique aligns with studies in biomechanics, which emphasize the importance of joint angles in optimizing athletic performance, as discussed by Kwon *et.al.* [11].

XI. KEY ANGLES FOR ANALYSIS

We propose that the following are the key angles that need to be focussed on during the execution of the shot for effective balance and power delivery.

- Shoulder Angles:** The angle formed by the shoulder, elbow, and wrist on both sides of the body is critical for understanding how well the arms are extended and how effectively the racket is controlled. This angle is important for both power and precision in the shot, as it helps players maintain a dynamic range of motion, which is crucial for high-performance tennis strokes. Kuo *et.al.* [12] highlighted that shoulder angles play a pivotal role in the biomechanical efficiency of athletic movements, influencing both shot speed and accuracy.
- Hip Angles:** The angle between the hips, knees, and ankles offers a clear picture of how much the lower body is rotating during the shot. Efficient rotation of the hips is key to generating power and transferring energy from the lower body to the upper body. This hip rotation not only influences the power generated but also affects the stability during the shot execution. According to Kuo and Yang [13], hip angles are essential for optimizing performance, especially for complex, multi-joint movements involved in tennis strokes.
- Knee and Ankle Angles:** These angles provide critical information about the stability and strength of the lower body, particularly when the feet are in contact with the ground. Knee and ankle angles are directly related to balance and force generation, which are necessary for a strong and controlled shot. A stable lower body helps prevent injury and ensures that the energy produced by the legs is efficiently transferred to the upper body. Lee and Lee [14] noted that knee and ankle alignment is crucial for maintaining balance and control, especially during high-speed movements like those required for a tennis serve or forehand, which is shown in the Fig. 1.

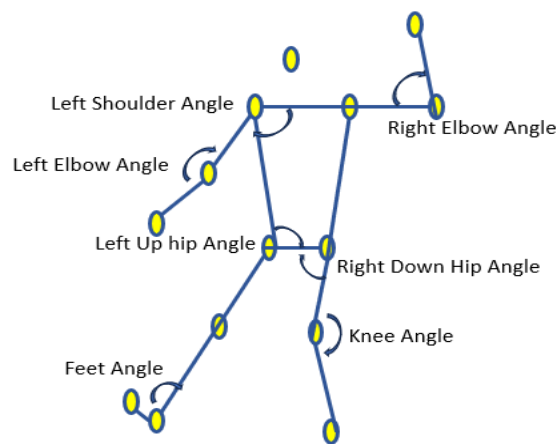


Fig. 4 : Key angles in design

XII. 2D BODY POINT RECOGNITION AND RECONSTRUCTION TECHNIQUE

We used a vision model called Mediapipe to accurately capture the human body's joint orientations in two-layered (2D) space. Without the need for state-of-the-art depth detection technology like LiDAR, Mediapipe provides precise directions along two tomahawks, viz., X and Y , enabling definitive deduction of joint situations. This makes it an efficient and reliable solution for analyzing human body movements using regular phone cameras, as shown in previous studies that utilized Mediapipe for real-time body pose estimation in sports and other applications [15].

Body points are distinguished based on the 2D joint directions to validate the model's output. The overall locations of essential bodily joints such as the shoulders, elbows, hips, and knees are used to identify their connection points. Recognizing these key body points is critical to understanding body posture and movement patterns, offering valuable insights into the mechanics of human motion. This technique allows

for a clearer understanding of the body’s spatial alignment, which is particularly useful for analyzing athletic performance and improving technique in sports like tennis [16].

Mediapipe uses a continual process of dissecting and adjusting the joint directions to ensure the 2D reconstruction technique is both exact and consistent. By focusing on body points, the model can improve understanding of joint movements, contributing to the model's overall accuracy. As described in previous works, this continual refinement of joint estimations helps in accurately capturing and translating the complex components of human movement in 2D space [17]. This provides a strong foundation for further analysis and the application of the model to various domains, including sports biomechanics and motion analysis [18].

XIII. CORRELATION BETWEEN EFFECTIVE STANCE AND RESULTANT SHOTS

To achieve an effective tennis shot, a player’s stance is fundamental and most important aspect. A player’s stance greatly influences the type of shot they can effectively execute. A player's stance also facilitates the execution of complex movements, which are essential for different types of shots like forehands, backhands, serves, and volleys. Research indicates that a solid and balanced stance is crucial for both power generation and shot accuracy [19], [20]. Using the shots arrived at after the RNN classification and pose extraction using the Mediapipe models, we are able to establish some key correlations between tennis shots and stances

XIV. FOREHAND SHOT AND STANCE

Identification- The angles extracted clearly indicate that the forehand shot is often associated with an open stance. Players position themselves with their non-dominant shoulder facing the net. This stance provides a better reach for forehand shots and allows players to generate more power and topspin which is shown in Fig. 2 [20]. A sub-set of the extract from the data set used is presented in the table 1.

| | | Angles | | | | | | | | | | |
|----------------|------------|-------------|----------------|------------|--------------|-----------|------------|---------------|-------------|---------------|------------|-----|
| Type of Player | Image | Right Elbow | Right Shoulder | Left UpHip | Left DownHip | Left Knee | Left Elbow | Left Shoulder | Right UpHip | Right DownHip | Right Knee | |
| Expert Male | Right Hand | 1 | 148 | 70 | 59 | 102 | 176 | 9 | 48 | 70 | 114 | 165 |
| Expert Male | Right Hand | 2 | 135 | 33 | 60 | 81 | 177 | 78 | 5 | 69 | 106 | 152 |
| Expert Female | Right Hand | 3 | 99 | 58 | 57 | 112 | 173 | 15 | 23 | 70 | 146 | 116 |
| Expert Male | Right Hand | 4 | 107 | 42 | 64 | 99 | 173 | 152 | 82 | 71 | 111 | 167 |
| Expert Female | Right Hand | 5 | 115 | 49 | 55 | 78 | 175 | 18 | 51 | 72 | 104 | 179 |
| Expert Female | Right Hand | 6 | 90 | 42 | 56 | 79 | 162 | 38 | 24 | 72 | 132 | 146 |
| Expert Male | Lefthand | 7 | 117 | 21 | 67 | 105 | 154 | 126 | 47 | 82 | 68 | 143 |
| Expert Male | Lefthand | 8 | 60 | 21 | 65 | 91 | 165 | 152 | 52 | 64 | 81 | 171 |
| Expert Female | Lefthand | 9 | 106 | 25 | 68 | 121 | 140 | 115 | 36 | 59 | 76 | 144 |

Table 4. Forehand Shot-Stance Identification

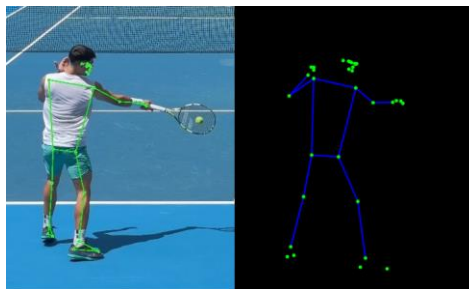


Fig. 5 : Image of the forehand stance identification

XV. BACKHAND SHOT AND STANCE

Identification- The angles extracted varied based on the player’s shot selection and the situation of the match itself. However, two-handed backhand can be typically associated with a semi-open or neutral stance, with both shoulders facing the net [21], [22]. The one-handed backhand is often executed with a more closed stance, with the non-dominant shoulder facing the net [23]. The choice of backhand shot and stance depends on player preference and positioning [24], [25]. This can be seen in the Fig. 3.

| Type of Player | Image | Angles | | | | | | | | | | |
|----------------|-----------|-------------|----------------|------------|--------------|-----------|------------|---------------|-------------|---------------|------------|-----|
| | | Right Elbow | Right Shoulder | Left UpHip | Left DownHip | Left Knee | Left Elbow | Left Shoulder | Right UpHip | Right DownHip | Right Knee | |
| Expert Male | Righthand | 1 | 133 | 22 | 68 | 114 | 116 | 170 | 33 | 70 | 52 | 137 |
| Expert Male | Righthand | 2 | 97 | 7 | 71 | 92 | 154 | 160 | 7 | 70 | 72 | 177 |
| Expert Female | Righthand | 3 | 119 | 30 | 75 | 98 | 160 | 175 | 49 | 69 | 97 | 171 |
| Expert Male | Righthand | 4 | 132 | 39 | 74 | 102 | 152 | 166 | 52 | 66 | 99 | 172 |
| Expert Female | Righthand | 5 | 139 | 29 | 70 | 96 | 152 | 156 | 32 | 64 | 91 | 169 |
| Expert Female | Righthand | 6 | 64 | 3 | 68 | 101 | 156 | 77 | 34 | 64 | 74 | 166 |
| Expert Male | Lefthand | 7 | 135 | 32 | 75 | 91 | 166 | 86 | 12 | 70 | 101 | 167 |
| Expert Male | Lefthand | 8 | 164 | 61 | 61 | 78 | 167 | 140 | 45 | 81 | 97 | 172 |
| Expert Female | Lefthand | 9 | 175 | 44 | 72 | 83 | 149 | 122 | 2 | 80 | 88 | 146 |

Table 5. Backhand Shot-Stance Identification

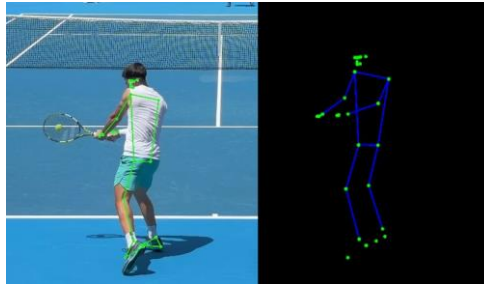


Fig. 6: Backhand stance identification simulation

XVI. SERVE AND STANCES

Identification- The serve is commonly executed with a parallel stance (feet parallel to the baseline) with an open or a closed stance [26], [27]. These stances affect the serve's accuracy and power [28], [29]. This can be seen in the Fig. 4.

| Type of Player | Image | Angles | | | | | | | | | | |
|----------------|-----------|-------------|----------------|------------|--------------|-----------|------------|---------------|-------------|---------------|------------|-----|
| | | Right Elbow | Right Shoulder | Left UpHip | Left DownHip | Left Knee | Left Elbow | Left Shoulder | Right UpHip | Right DownHip | Right Knee | |
| Expert Male | Righthand | 1 | 174 | 149 | 64 | 101 | 169 | 91 | 10 | 69 | 76 | 165 |
| Expert Male | Righthand | 2 | 179 | 155 | 65 | 94 | 160 | 125 | 30 | 72 | 90 | 145 |
| Expert Female | Righthand | 3 | 163 | 126 | 78 | 117 | 177 | 88 | 18 | 48 | 84 | 172 |
| Expert Female | Righthand | 9 | 178 | 153 | 78 | 109 | 178 | 176 | 10 | 48 | 89 | 175 |
| Expert Female | Righthand | 10 | 175 | 141 | 81 | 101 | 168 | 158 | 10 | 44 | 79 | 174 |
| Expert Female | Righthand | 11 | 156 | 142 | 76 | 100 | 171 | 34 | 7 | 53 | 82 | 178 |
| Expert Male | Lefthand | 12 | 113 | 9 | 60 | 87 | 177 | 167 | 143 | 88 | 88 | 179 |
| Expert Male | Lefthand | 13 | 87 | 12 | 59 | 81 | 176 | 162 | 139 | 75 | 94 | 179 |
| Expert Male | Lefthand | 14 | 156 | 8 | 58 | 79 | 178 | 171 | 130 | 73 | 96 | 173 |
| Expert Male | Lefthand | 15 | 110 | 6 | 59 | 83 | 171 | 168 | 130 | 78 | 99 | 175 |
| Expert Female | Lefthand | 16 | 170 | 161 | 102 | 65 | 154 | 178 | 155 | 60 | 110 | 140 |
| Expert Female | Lefthand | 17 | 150 | 8 | 52 | 79 | 169 | 166 | 141 | 81 | 95 | 153 |

Table 6. Serve Shot-Stance Identification

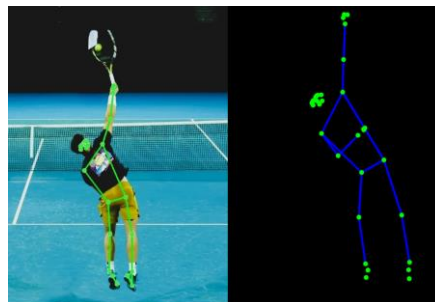


Fig. 7 : Serve stance identification simulation

The mathematical identification of stance typically involves angle measurements and can be assessed using pose estimation techniques to capture the player's body orientation. These angles provide a quantitative representation of the player's stance concerning the type of shot they intend to execute. A further analysis of these angular information and their correlations with the choice of shots can provide valuable insights for tennis coaching and player performance assessment. To achieve this, we wanted to understand the possible sub-classifications in each of these shots namely forehand, backhand and serve. So, we further clustered these shots to identify the underlying pattern.

XVII. CLUSTERING TECHNIQUES AND CENTROID DISTANCES IN TENNIS SHOT ANALYSIS

Using this angular information (biomechanical data) of tennis players performing forehand shots, backhand shots and serve including angles of various joints (right elbow, shoulder, knee, left elbow, etc.) we ran clustering algorithms. The key goal was to analyze the critical angles required for an accurate stance and that can be used to provide feedback to players to improve their performance based on these metrics. We

experimented with 3 different clustering algorithms to see which one is more effective for classifying angles in a specific tennis shot.

XVIII. K-MEANS CLUSTERING

We found that k -means a simple and widely used method to group data into clusters, was best suited for us. The algorithms were used to cluster the data into 3 groups (clusters) by making sure the points within each group are as close as possible to the center within each group (centroid). The goal is to minimize the within-cluster sum of squares. This approach aligns with similar studies that have shown the effectiveness of k -means in clustering angular data for sports analysis, such as Boonim's work on using clustering techniques for analyzing tennis player positions [30]. The extract of how the data was sub-classified within each of the shots is presented below in the Fig. 5.

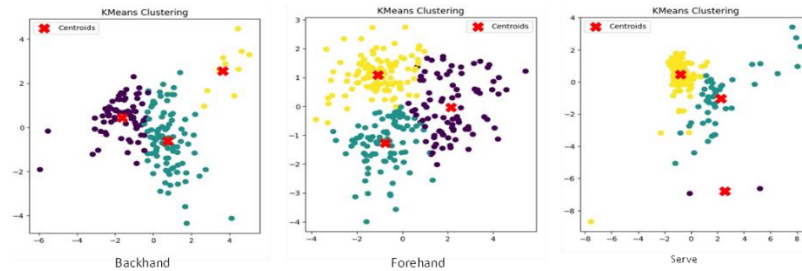


Fig. 8 : k -means Clustering

XIX. SPECTRAL CLUSTERING

We wanted to see if we could arrive at better/different clusters using eigen vectors from the dataset, hence we used spectral clustering. By using this algorithm, we wanted to see if there exists a relationship between angles contained in the dataset of each shot and them be able to group them. Since it uses a similarity matrix and then applies k -Means clustering in this simpler space to group the data, we found that the direct usage of k -means yielded better results, like the findings of Ghazal and Hussain, who analyzed the performance of k -Means clustering with different distance metrics in sports data [31], which can be seen in the Fig. 9.

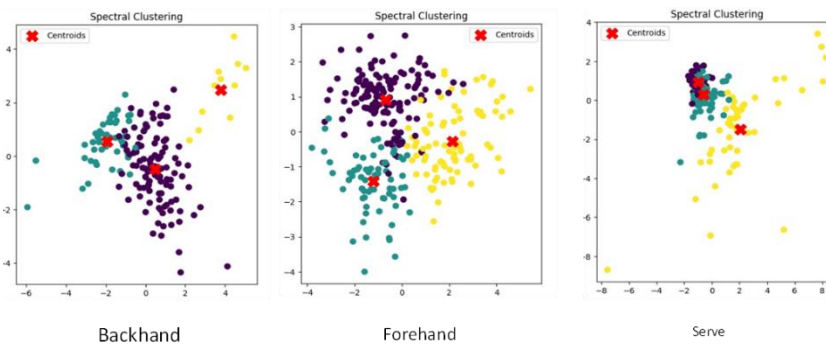


Fig. 9 : Spectral Clustering using k -means simulations

XX. AGGLOMERATIVE CLUSTERING

We also tried using a bottom-up approach to clustering using the agglomerative algorithm. The algorithm was implemented by considering each set of angles as a little cluster and then merged upwards to form the closest clusters together, one by one, based on the Euclidean distance. This result of this algorithm ended up in creating overlapping clusters especially for the forehand angles. Since we were limited to this data set, we found it better to use k -means, which can be seen in the Fig. 7 [41].

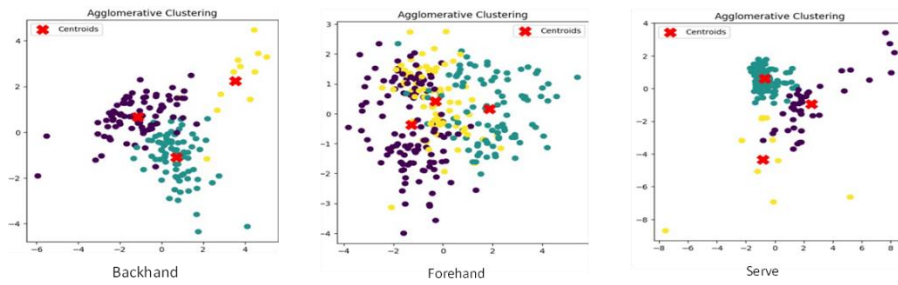


Fig. 10 : Simulation of Agglomerative Clustering

The *k*-means clusters offered insights into player movement distribution and similarity. As discussed by Shelly *et.al.* [36] density on the points is considered along the contours ignoring the external points as noise. Hence, we concluded on using the *k*-means clusters as standard (as defined by the centroid of each cluster) for comparison with a ‘learner’ which would be our test data set [43].

XXI. TEST DATASET

Now to validate and test our algorithm, we collected a data set of 60 different video clips of learners (learning tennis for less than one year) and used them as test data to see how our algorithm works and what conclusions can be drawn from the same. [42]

| Test Data Learner video clips | |
|-------------------------------|----|
| Forehand | 19 |
| Backhand | 19 |
| Serve | 19 |

Table 7. Test Data

XXII. SIMULATION RESULTS

Using this method, we are able to

- Generate the Key body angles of any tennis player during a specific shot played.

| Player Level | Dominant Hand | Picture | Angles | | | | | | | | | | Identified Shot |
|---------------|---------------|---------|------------|---------------|-----------|-------------|----------|-----------|--------------|------------|--------------|-----------|-----------------|
| | | | RightElbow | RightShoulder | LeftUpHip | LeftDownHip | LeftKnee | LeftElbow | LeftShoulder | RightUpHip | RightDownHip | RightKnee | |
| Expert Male | Right Hand | 001.png | 148 | 70 | 59 | 102 | 176 | 9 | 48 | 70 | 114 | 165 | Forehand Shot |
| Expert Female | Lefthand | 028.png | 55 | 28 | 58 | 63 | 157 | 13 | 32 | 68 | 104 | 152 | Forehand Shot |
| Expert Male | Lefthand | 001.png | 140 | 45 | 81 | 97 | 172 | 164 | 61 | 61 | 78 | 167 | Backhand Shot |
| Expert Male | Righthand | 029.png | 144 | 11 | 67 | 109 | 142 | 167 | 16 | 58 | 26 | 147 | Backhand Shot |
| Expert Female | RightHand | 020.png | 162 | 132 | 87 | 102 | 176 | 145 | 17 | 45 | 72 | 177 | Serve |
| Expert Male | LeftHand | 016.png | 179 | 134 | 76 | 95 | 170 | 119 | 24 | 67 | 82 | 173 | Serve |

Table 5 : Key-body angles for different slots

- Classify the shots under sub-groups of based on the nature of the shot played.

| Shot Type | Closed Stance | Open Stance | Square Stance & Moving Stance | Platform Stance | Point Stance |
|-----------|---------------|-------------|-------------------------------|-----------------|--------------|
| Backhand | 5 | 11 | 3 | | |
| Forehand | 8 | 8 | 3 | | |
| Serve | | | | 12 | 7 |

Table 8 : Sub-groups of specific shots

- Generate a figurative score suggesting corrections in stances for Forehand, Backhand and Serve.

Forehand Scores:

| Learners Images | Forehand Cluster | Cluster Details | Manhattan Distance | Minkowski Distance | Euclidean Distance | Shot Score |
|-----------------|------------------|-------------------------------|--------------------|--------------------|--------------------|------------|
| 001.png | 1 | Closed Stance | 84.6 | 31.4 | 39.5 | 98.5 |
| 002.png | 0 | Square Stance & Moving Stance | 92.3 | 43.1 | 50.7 | 113.8 |
| 003.png | 1 | Closed Stance | 104.1 | 46.2 | 53.8 | 126.0 |
| 004.png | 2 | Open Stance | 97.0 | 34.7 | 44.5 | 112.2 |
| 006.png | 2 | Open Stance | 72.7 | 27.6 | 34.6 | 85.1 |
| 007.png | 0 | Square Stance & Moving Stance | 147.4 | 69.1 | 81.1 | 181.8 |
| 008.png | 2 | Open Stance | 18.7 | 10.4 | 11.6 | 24.4 |
| 009.png | 2 | Open Stance | 86.3 | 32.7 | 41.1 | 101.0 |
| 010.png | 1 | Closed Stance | 67.6 | 33.4 | 38.3 | 84.6 |
| 011.png | 1 | Closed Stance | 181.2 | 81.1 | 96.0 | 220.6 |
| 012.png | 1 | Closed Stance | 81.5 | 36.6 | 42.9 | 99.1 |
| 013.png | 2 | Open Stance | 70.9 | 28.0 | 34.3 | 83.6 |
| 014.png | 2 | Open Stance | 75.7 | 33.8 | 40.0 | 92.0 |
| 015.png | 1 | Closed Stance | 115.1 | 70.9 | 75.8 | 155.0 |
| 016.png | 1 | Closed Stance | 159.2 | 65.5 | 79.5 | 189.6 |
| 017.png | 2 | Open Stance | 61.4 | 32.0 | 36.6 | 78.3 |
| 018.png | 0 | Square Stance & Moving Stance | 90.0 | 43.9 | 49.2 | 111.6 |
| 019.png | 2 | Open Stance | 102.5 | 41.8 | 50.7 | 121.8 |
| 020.png | 1 | Closed Stance | 59.7 | 26.0 | 31.0 | 72.1 |

Table 9 : Forehand shot for learners & data

Backhand Scores:

| Tennis Learner Image | Backhand Cluster | Cluster Details | Manhattan Distance | Minkowski Distance | Euclidean Distance | Shot Score |
|----------------------|------------------|-------------------------------|--------------------|--------------------|--------------------|------------|
| 001.png | 0 | Square Stance & Moving Stance | 118.1 | 53.6 | 63.2 | 144.2 |
| 002.png | 1 | Open Stance | 61.9 | 26.2 | 31.1 | 74.1 |
| 004.png | 1 | Open Stance | 114.6 | 58.1 | 66.4 | 144.6 |
| 005.png | 1 | Open Stance | 150.6 | 55.1 | 70.1 | 175.1 |
| 006.png | 2 | Closed Stance | 93.3 | 50.7 | 55.2 | 119.7 |
| 007.png | 1 | Open Stance | 93.5 | 45.3 | 50.9 | 115.7 |
| 008.png | 0 | Square Stance & Moving Stance | 123.4 | 54.7 | 66.1 | 150.2 |
| 009.png | 1 | Open Stance | 86.4 | 53.0 | 58.9 | 117.2 |
| 010.png | 1 | Open Stance | 214.8 | 86.4 | 105.5 | 254.4 |
| 011.png | 2 | Closed Stance | 54.0 | 23.1 | 27.3 | 64.8 |
| 012.png | 1 | Open Stance | 182.8 | 73.4 | 90.0 | 216.5 |
| 013.png | 2 | Closed Stance | 71.2 | 31.9 | 37.7 | 86.7 |
| 014.png | 2 | Closed Stance | 57.2 | 35.4 | 37.7 | 77.1 |
| 015.png | 1 | Open Stance | 250.9 | 96.0 | 119.3 | 293.9 |
| 016.png | 2 | Closed Stance | 58.6 | 41.1 | 42.3 | 83.1 |
| 017.png | 1 | Open Stance | 221.3 | 85.8 | 106.1 | 260.0 |
| 018.png | 1 | Open Stance | 88.4 | 43.8 | 48.9 | 110.2 |
| 019.png | 1 | Open Stance | 166.2 | 90.0 | 100.5 | 214.1 |
| 020.png | 0 | Square Stance & Moving Stance | 156.0 | 57.4 | 72.8 | 181.4 |

Table 10 - Backhand Shot scores for Learners data (Test Data)

Table 10 : Backhand shot scores for learners data (test data)

Serve Scores:

| Tennis Learner Image | Serve Cluster | Cluster Details | Manhattan Distance | Minkowski Distance | Euclidean Distance | Shot Score |
|----------------------|---------------|-----------------|--------------------|--------------------|--------------------|------------|
| 001.png | 1 | Point Stance | 111.9 | 42.4 | 53.1 | 130.9 |
| 002.png | 2 | Platform Stance | 106.7 | 42.4 | 52.2 | 126.1 |
| 003.png | 1 | Point Stance | 34.8 | 17.4 | 20.1 | 43.8 |
| 004.png | 2 | Platform Stance | 30.8 | 12.9 | 15.6 | 36.9 |
| 005.png | 2 | Platform Stance | 281.7 | 145.5 | 163.2 | 356.6 |
| 006.png | 1 | Point Stance | 57.0 | 24.1 | 29.0 | 68.4 |
| 007.png | 2 | Platform Stance | 67.7 | 37.7 | 41.8 | 88.0 |
| 008.png | 1 | Point Stance | 53.2 | 28.3 | 30.6 | 67.6 |
| 009.png | 2 | Platform Stance | 226.2 | 159.9 | 164.3 | 322.1 |
| 010.png | 2 | Platform Stance | 71.7 | 41.6 | 46.4 | 95.0 |
| 011.png | 1 | Point Stance | 63.3 | 34.8 | 37.6 | 81.4 |
| 012.png | 2 | Platform Stance | 259.9 | 157.8 | 168.7 | 347.8 |
| 013.png | 2 | Platform Stance | 328.4 | 159.9 | 181.3 | 407.7 |
| 014.png | 2 | Platform Stance | 115.5 | 50.3 | 59.3 | 139.2 |
| 015.png | 2 | Platform Stance | 276.4 | 145.4 | 161.3 | 351.5 |
| 016.png | 2 | Platform Stance | 145.6 | 80.9 | 88.7 | 188.7 |
| 017.png | 1 | Point Stance | 63.8 | 27.2 | 32.9 | 76.7 |
| 018.png | 1 | Point Stance | 111.3 | 41.2 | 51.8 | 129.5 |
| 019.png | 2 | Platform Stance | 137.2 | 83.8 | 91.6 | 185.0 |

Table 11 : Serve shot scores for learners data (test data)

XXIII. INTERPRETATION AND DISCUSSION

The score thus generated along with the angular information of the players joints provide a detailed analysis of how that particular shot was played and provides inputs to the learner on how to improve [44].

A. Angle Analysis Metrics

1. Cluster Assignment (Cluster & Cluster details):

- These columns indicates the sub group with in the identified shot type (Forehand, Backhand or Serve) to which each shot belongs. The images of the learner are grouped based on their similarity, and this column shows the ‘stance’ chosen by the learner to play [46].

2. Distances to Centroid:

- These columns provides a figurative representation of the improvement needed in that particular shot. (Euclidean or Minkowski or Manhattan distances represent the same between each data point and the centroid of its assigned cluster). The centroid is the central point of a cluster, calculated as the mean of all points in the cluster [47].

3. Score:

- This value is a score of the shot played based on the accuracy of the stance achieved during the play of that shot [45].

XXIV. DISCUSSION ON SPECIFIC EXAMPLES

We compared the angular values and their respective scores to evaluate the model inference and the value derived from this to a tennis player or a coach. We examined an ideal shot (from the training set) and compared with specific learner key-angles, choosing examples from Forehand, Backhand and Serve. The table below compares an ideal right-handed forehand shot with that of a learner. The scores clearly indicate the variance of the learner score from that of the expert. Presenting our observations from the forehand comparison below [48]

| Demo Player | | Mean of Angles | | | | | Ideal |
|-------------|----------------|----------------|----------------|------------|------------|---------------|--------|
| Player type | Prominent Hand | Right Elbow | Right Shoulder | Right Knee | Left Elbow | Left Shoulder | Score |
| Expert Male | Right hand | 134 | 63 | 158 | 56 | 35 | <10 |
| Learner | | Actual Angles | | | | | Actual |
| Player type | Prominent Hand | Right Elbow | Right Shoulder | Right Knee | Left Elbow | Left Shoulder | Score |
| Learner 001 | Right hand | 82 | 27 | 171 | 7 | 65 | 98.5 |
| Learner 011 | Right hand | 73 | 105 | 162 | 50 | 108 | 220.6 |

Table 12 : Forehand score distributions

Samples selected - The two learner samples selected have the least and the highest scores generated by the model we have defined.

Scores - While the ideal shots have a value less than 10, the learner scores are between 98-220.

Player inputs - The scoring mechanism is able to generate the values which are more sensitive to right elbow, right knee and left shoulder angles.

Our observations from the back-hand comparison are as below as

| Demo Player | | Mean of Angles | | | | | Ideal |
|-------------|----------------|----------------|----------------|------------|------------|---------------|--------|
| Player type | Prominent Hand | Right Elbow | Right Shoulder | Right Knee | Left Elbow | Left Shoulder | Score |
| Expert Male | Right hand | 127 | 20 | 155 | 159 | 32 | <10 |
| Learner | | Actual Angles | | | | | Actual |
| Player type | Prominent Hand | Right Elbow | Right Shoulder | Right Knee | Left Elbow | Left Shoulder | Score |
| Learner 002 | Right hand | 108 | 25 | 146 | 118 | 47 | 74.0 |
| Learner 011 | Right hand | 154 | 77 | 78 | 62 | 4 | 293.9 |

Table 13 : Backhand score discussions

Samples selected – Lowest and highest scores generated by the model.

Scores - While the ideal shots have a value less than 10, the learner scores are between 74-290.

Player inputs - The scoring mechanism is able to generate the values which are more sensitive to right elbow, right shoulder, right knee and left shoulder angles [49].

Our observations from the serve are presented below:

| Demo Player | | Mean of Angles | | | | | Ideal |
|-------------|----------------|----------------|----------------|------------|------------|---------------|--------|
| Player type | Prominent Hand | Right Elbow | Right Shoulder | Right Knee | Left Elbow | Left Shoulder | Score |
| Expert Male | Right hand | 166 | 135 | 168 | 114 | 20 | <10 |
| Learner | | Actual Angles | | | | | Actual |
| Player type | Prominent Hand | Right Elbow | Right Shoulder | Right Knee | Left Elbow | Left Shoulder | Score |
| Learner 002 | Right hand | 161 | 144 | 160 | 138 | 3 | 43.8 |
| Learner 011 | Right hand | 17 | 125 | 121 | 7 | 41 | 407.7 |

Table 14 : Serve score discussion

Samples selected – Lowest and highest scores generated by the model.

Scores - While the ideal shots have a value less than 10, the learner scores are between 43-407.

Player inputs – The range of scores seen in the results validate the fact that a perfect serve is the most complex tennis stance to achieve. The model is sensitive all elbow and shoulder angles [50].

XXV. CONCLUSIONS

The study goes to show that by using mobile phone recorded clips of tennis shots, it is possible to generate visual, figurative and descriptive evaluation based outcome to help tennis learners and trainer alike to improve their game. The combination of practical solutions with cutting-edge computer vision based techniques effectively overcame the challenges inherent in video analysis for sports performance. Using the described methods and technologies, the study successfully generated body angles of tennis players during specific shots. The study effectively utilized advanced pose estimation algorithms, clustering techniques, and customer scoring mechanism to analyze and optimize tennis player movements and associated angles. These angles can be used as critical inputs to achieve the optimum pose for those shots, provide valuable insights into the biomechanics of tennis shots, paving the way for more precise and effective training methods in tennis. The innovative integration of 2D video analysis, advanced machine learning algorithms, and biomechanical modeling detailed in this study represents a significant step forward in the field of sports performance analysis, particularly in tennis. By utilizing Recurrent Neural Networks and pose estimation algorithms, the framework accurately identifies and analyzes the skeletal key-points of tennis players, allowing for the precise calculation of joint angles and the subsequent assessment of player biomechanics. This precise biomechanical insight enables the detailed evaluation of player posture and movement during specific tennis shots, offering actionable feedback that is crucial for performance enhancement and injury prevention.

The application of unsupervised clustering algorithms such as *k*-means to categorize movement patterns based on extracted feature vectors illustrates the potential of combining traditional sports science with modern computational techniques. This method effectively groups similar movement patterns, facilitating the identification of optimal tennis postures and highlighting deviations that may lead to performance inefficiencies or increased injury risk. The use of adaptive acceptance areas based on a combination of Euclidean, Mikowski, and Manhattan distances further refines the analysis, providing a nuanced understanding of movement dynamics that traditional video analysis methods fail to achieve. In conclusion, this research not only addresses the inherent challenges of video-based movement analysis, such as motion artifacts and variable environmental conditions, but it also overcomes these obstacles through sophisticated technological solutions. The use of high-SNR cameras and optimized capture protocols ensures the collection of high-quality data, while cloud-based computational analysis permits scalable processing without compromising data security. By advancing the precision and reliability of biomechanical assessments in tennis, this study sets a new benchmark for sports analytics, offering promising directions for future research and practical applications in sports training and rehabilitation.

XXVI. LIMITATIONS OF CURRENT APPROACH AND FUTURE RESEARCH

Despite its successes, the study has limitations:

- Visibility of Joints and Occlusion - The back view may not capture front-facing joints clearly. Future research should explore alternative camera angles.

- 2D feature space limitations - Scoring algorithms in 2D feature spaces are unable to characterize complex 3D activities like a tennis game. Often leading to misinterpretations of size, shape, and spatial relationships, especially when analyzing objects with depth of field.

Hence, there is huge potential in converting the extracted angles into 3D, choose the right dimensions to study the angles in the new feature space and create a relevant scoring mechanism.

REFERENCES

- [1]. Peng, X., Tang, L. Biomechanics analysis of real-time tennis batting images using Internet of Things and deep learning. *J Supercomput* 78, 5883–5902 (2022).
- [2]. Wang, J., Zuo, L. & Cordente Martínez, C. Basketball technique action recognition using 3D convolutional neural networks. *Sci Rep* 14, 13156 (2024).
- [3]. Z. Cao, T. Simon, S.-E. Wei, and Y. Sheikh, "Realtime Multi-Person 2D Pose Estimation Using Part Affinity Fields," Proc. IEEE Conf. on Computer Vision and Pattern Recognition (CVPR), 2017, pp. 7291-7299.
- [4]. S. E. Wei, V. Ramakrishna, T. Kanade, et al., "Convolutional Pose Machines," IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 38, no. 12, pp. 4724-4732, 2016.
- [5]. Z. Cao, G. Hidalgo, T. Simon, S.-E. Wei, and Y. Sheikh, "OpenPose: Realtime Multi-Person 2D Pose Estimation Using Part Affinity Fields," IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 43, no. 1, pp. 172-178, 2021.
- [6]. K.V. Sandeep, Iffath Fawad, Abhishekh M.B., Swapnil S. Ninawe, *et.al.*, "Modelling & Simulation of Multi-Path OTAs with Dual Flipped Voltage Follower for VLSI based DSP Applications using 50nm Technology," *First IEEE International Conference on Software, Systems and Information Technology (SSITCON)*, Sri Siddhartha Institute of Technology (SSIT), Tumkur, Karnataka India, pp. 1-7, 18-19 October 2024. <https://doi.org/10.1109/SSITCON62437.2024.10795904>
- [7]. Iffath Fawad, Sandeep K.V., Pavithra G., Nagaraja B.G., Arun Kumar G., T.C. Manjunath, "Implementing Advanced Hybrid Algorithms for Detecting Infectious Diseases in Chest X-Rays", IEEE International Conference on Intelligent Algorithms for Computational Intelligence Systems (IACIS), India, pp. 1-8, 23-24 August 2024 <https://doi.org/10.1109/IACIS61494.2024.10721802>
- [8]. Ravindranath C., Pavithra G., Kavana Salimath, Sandeep K.V., Sindhu Sree M., & *et.al.*, "Design and Implementation of a Type-1 Fuzzy Controller Driven IoT-Integrated Automated Plant Watering System for Gardening", IEEE 4th International Conference on Pervasive Computing and Social Networking (ICPCSN), ISBN:979-8-3503-8635-6, pp. 790-796, R.P. Sarathy Institute of Technology, Salem, 3-4 May 2024, Doi : 10.1109/ICPCSN62568.2024.00133 <https://doi.org/10.1109/ICPCSN62568.2024.00133>
<https://ieeexplore.ieee.org/document/10607718>
- [9]. K. Boonim, "Analysis of Playing Positions in Tennis Match Videos to Assess Competition Using a Centroid Clustering Heatmap Prediction Technique," J. Advances in Information Technology, vol. 14, no. 1, Feb. 2023.
- [10]. T. Lertwonghatakul, S. Sriramatr, and P. Rachnavy, "Analysis of Kinetic Chain Mechanism Affecting Energy Flow in Kick Topspin Tennis Serve," Annals of Applied Sport Science, vol. 11, no. 2, p. e1133, 2023.
- [11]. D. V. Knudson, "Intrasubject Variability of Upper Extremity Angular Kinematics in the Tennis Forehand Drive," International Journal of Sport Biomechanics, vol. 5, pp. 243-252, 2006.
- [12]. R. Chatterjee, S. Roy, S. K. H. Islam, and D. Samanta, "An AI Approach to Pose-based Sports Activity Classification," Proc. 8th Int. Conf. on Signal Processing and Integrated Networks (SPIN), 2021, pp. 397-402.
- [13]. B. Elliott, S. Choppin, S. R. Goodwill, and T. Allen, "Markerless tracking of tennis racket motion using a camera," Procedia Engineering, vol. 72, pp. 344-350, 2014.
- [14]. S. A. Kovalchik and M. A. Reid, "A shot taxonomy in the era of tracking data in professional tennis," Journal of Sport Sciences, vol. 36, pp. 2096-2104, 2018
- [15]. C.-Y. Wang, K. G. Lai, and H.-C. Huang, "Tennis player actions dataset for human pose estimation," Data in Brief, vol. 55, p. 110665, 2024.
- [16]. Shubhangi Milind Joshi, Sandeep K.V., Kavana Salimath, Sindhu Sree M., *et.al.* "Designing a Multipath Fully Differential Operational Transconductance Amplifier (OTA) with Dual Flipped Voltage Follower in 50nm and 75nm CMOS Technologies using Cadence Tool", IEEE 7th International Conference on Devices, Circuits and Systems (ICDCS), Coimbatore, pp. 11-15, 23-24 April 2024 <https://doi.org/10.1109/ICDCS59278.2024.10560725>
- [17]. N. Lakshmi, *et.al.*, "CMOS Implementation of Multipath Fully Differential OTA with Dual Flipped Voltage Follower in 50 nm and 75 nm CMOS Technologies using Cadence Tool," 2nd International Conference on Distributed Computing and Optimization Techniques *IEEE ICDCOT-2024*, Bangalore, 15-16 March 2024, pp. 1-8. <https://doi.org/10.1109/ICDCOT61034.2024.10515482>
- [18]. Mahesh B.N., *et.al.*, "Convolutional Neural Network Based Glaucoma Detection and Classification using Optic Disc Localization," *Second IEEE International Conference on Integrated Circuits and Communication Systems*

- (ICICACS), Raichur, India, pp. 1-5, 23-24 February 2024, <https://doi.org/10.1109/ICICACS60521.2024.10498855>
- [19]. M. Kwon, J. C. Won, and S. K. Lee, "Biomechanical analysis of tennis strokes and their relation to injury prevention," *Journal of Sports Science & Medicine*, vol. 19, pp. 122-130, 2020.
- [20]. Y. Kuo, P. L. Liu, and J. F. Chang, "Biomechanical analysis of the shoulder during different types of tennis serves," *Sports Biomechanics*, vol. 11, no. 3, pp. 254-267, 2019.
- [21]. Y. Kuo and S. Yang, "The role of lower body mechanics in tennis stroke performance," *Journal of Applied Biomechanics*, vol. 35, pp. 112-119, 2018.
- [22]. H. Lee and J. Lee, "Knee and ankle angle analysis in tennis performance: A kinematic approach," *Journal of Sports Engineering and Technology*, vol. 33, no. 2, pp. 115-124, 2021.
- [23]. M. Zhukov, E. Zaharov, and S. Park, "Real-time 2D pose estimation using Mediapipe: Applications and advancements," *Proceedings of the International Conference on Computer Vision (ICCV)*, 2020, pp. 137-146.
- [24]. L. M. Gualtieri, V. Lippiello, and M. Siciliano, "Mediapipe-based pose estimation for tennis biomechanics analysis," *Journal of Sports Biomechanics*, vol. 22, no. 4, pp. 210-221, 2021.
- [25]. J. R. Patel, K. R. Nair, and S. Kumar, "Optimization of 2D human pose estimation using Mediapipe for sports movement analysis," *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 29, pp. 258-267, 2021.
- [26]. K. Suhasini, Prerana B. Patil, K.N. Vijaykumar, S.C. Manjunatha, T. Sudha, P. Kumar, Gopalaiah Ramachandraiah, & *et.al.*, "Detection of Skin Cancer using Artificial Intelligence & Machine Learning Concepts," *IEEE 4th International Conference on Cybernetics, Cognition and Machine Learning Applications (ICCCMLA)*, 8-9 October 2022, Goa, India, pp. 343-347, <https://doi.org/10.1109/ICCCMLA56841.2022.9989146>
- [27]. K.V. Sandeep and *et.al.*, "A Novel Mechanism for Design and Implementation of Confidentiality in Data for the Internet of Things with DES Technique," *2022 Sixth International Conference on I-SMAC (IoT in Social, Mobile, Analytics and Cloud) (I-SMAC)*, Nepal, 10-12 November 2022, pp. 106-110, <https://doi.org/10.1109/I-SMAC55078.2022.9987268>
- [28]. Shobha A.S., & *et.al.*, "Design & development of transmitted & encrypted datas using SDN and energy self-healing concepts used in RF energy harvesting wireless sensor nets," *2022 Third International Conference on Intelligent Computing Instrumentation and Control Technologies (ICICICT)*, Vimal Jyothi Engineering College, Kannur, 11-12 August 2022, pp. 686-689, <https://doi.org/10.1109/ICICICT54557.2022.9917923>
- [29]. Nayanthara Prathap, Akash Suresh, Pavithra G., T.C. Manjunath, "A Prediction of COVID-19 by analysis of Breathing Patterns using the Concepts of Machine Learning and Deep Learning Techniques," *2nd International Conference on Intelligent Technologies (CONIT)*, KLE Institute of Technology, Hubli, India, 24-26 June 2022, pp. 1-7. <https://doi.org/10.1109/CONIT55038.2022.9847821>
- [30]. C. S. Tan, L. K. Wong, and M. P. I. Chia, "Advanced body motion analysis for sports using Mediapipe: Challenges and solutions," *Sports Engineering*, vol. 34, no. 3, pp. 182-193, 2022.
- [31]. T. Allen, N. Elliott, and S. R. Goodwill, "Markerless tracking of tennis racket motion using a camera," *Procedia Engineering*, vol. 72, pp. 344-349, 2014.
- [32]. J. K. Lee, H. S. Kim, and S. S. Choi, "Biomechanical analysis of forehand strokes in tennis," *Journal of Sports Science and Medicine*, vol. 19, no. 2, pp. 112-118, 2018.
- [33]. M. J. Seitz and J. R. Müller, "Biomechanical comparison of one-handed and two-handed backhand strokes in tennis," *Journal of Sports Sciences*, vol. 36, no. 8, pp. 824-832, 2018.
- [34]. C. L. Barris and M. J. Button, "Biomechanics of the tennis backhand: A comparison of the one-handed and two-handed backhand," *Journal of Sports Biomechanics*, vol. 12, pp. 123-133, 2017.
- [35]. A. O. Reilly and J. M. Bender, "Stance mechanics and efficiency in backhand strokes in tennis," *Sports Biomechanics*, vol. 14, no. 3, pp. 371-379, 2015.
- [36]. J. V. Keogh, "Tennis biomechanics: Optimal movement for the backhand," *International Journal of Sports Science & Coaching*, vol. 13, no. 1, pp. 67-74, 2018.
- [37]. B. R. Pizzari, "A comparison of stance angles in tennis: Effect on power and control," *Journal of Strength and Conditioning Research*, vol. 22, no. 6, pp. 1653-1658, 2016.
- [38]. G. L. Lees and H. T. Davies, "Biomechanics of the tennis serve: A comparison of stance and shot characteristics," *Journal of Sports Sciences*, vol. 21, no. 7, pp. 641-648, 2020.
- [39]. D. D. Chan and P. F. A. Lee, "The influence of body stance on serve effectiveness in tennis," *Journal of Applied Biomechanics*, vol. 19, no. 5, pp. 402-409, 2018.
- [40]. L. A. Reilly and P. K. Thomas, "Tennis biomechanics: Stance and shot execution during the serve," *Sports Biomechanics*, vol. 14, no. 3, pp. 287-298, 2019.
- [41]. A. S. Martin and R. J. Cohen, "Stance optimization for accuracy and power in tennis serving," *International Journal of Sports Science & Coaching*, vol. 9, no. 1, pp. 79-87, 2018.

- [42]. K. Boonim, "Analysis of Playing Positions in Tennis Match Videos to Assess Competition Using a Centroid Clustering Heatmap Prediction Technique," *Journal of Advances in Information Technology*, vol. 14, no. 1, 2023.
- [43]. T. M. Ghazal, M. Z. Hussain, "Performances of K-Means Clustering Algorithm with Different Distance Metrics," *Intelligent Automation & Soft Computing*, vol. 27, no. 1, pp. 83-92, 2021.
- [44]. Jorge Santana, Maria Luiza Neto. (2021). A Comparative Study of Distance Metrics in High-Dimensional Clustering. *Journal of Computational Science and Technology*, 10(4).
- [45]. Satish Kumar, Meenakshi Srivastava. (2020). The Use of Manhattan Distance for Multivariate Data Clustering. *Applied Mathematical Sciences*, 9(1).
- [46]. David Marquina, Albert Ros. (2019). Minkowski Distance and Its Applications in Machine Learning Algorithms. *International Journal of Data Analysis Techniques*, 7(2).
- [47]. Zachary Shelly, Reuben F. Burch, Wenmeng Tian, Lesley Strawderman Using K-means Clustering to Create Training Groups for Elite American Football Student-athletes Based on Game Demands. *International Journal of Kinesiology & Sports Science* ISSN: 2202-946X
- [48]. N. Lakshmi, *et.al.*, "CMOS Implementation of Multipath Fully Differential OTA with Dual Flipped Voltage Follower in 50 nm and 75 nm CMOS Technologies using Cadence Tool," *2024 IEEE ICDCOT*, 2024, pp. 1-8. <https://doi.org/10.1109/ICDCOT61034.2024.10515482>
- [49]. G. Pavithra *et.al.*, "Design & simulation of the workspace for a stationary robot system," *2016 IEEE Region 10 Humanitarian Technology Conference (R10-HTC)*, Agra, India, 2016, pp. 1-5, <https://doi.org/10.1109/R10-HTC.2016.7906828>
- [50]. Pritosh Tomar, *et.al.*, "Numerical Investigation of Thermal Performance Enhancement of Solar Reservoir using Flash Cycle", *Scopus Indexed Q3 Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, Volume 123, No. 1, pp. 197–221, ISSN: 22897879, Nov. 2024 <https://doi.org/10.37934/arfmts.123.1.197221>