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Adaptive Energy Harvesting for WSN: Optimizing Mechanical Vibration Conversion and OFDM Communication Efficiency

Abstract: Mechanical energy, widely available in environments suitable for Wireless Sensor Networks (WSNs), presents an attractive opportunity for powering sensors through ambient vibration energy harvesting. Vibration sources, such as human motion (frequencies below 10 Hz) and machinery vibrations (frequencies above 30 Hz), offer distinct energy profiles. In WSNs, communication often relies on Orthogonal Frequency Division Multiplexing (OFDM) due to its robustness against interference and propagation challenges. However, designing OFDM circuits under strict power constraints remains challenging when relying on energy harvested from ambient vibrations. In industrial environments, where vibration levels are typically low and frequencies match power line harmonics (50 or 60 Hz), resonant inertial generators are employed. These generators are limited by their narrow frequency range, prompting the development of adaptive kinetic energy harvesting devices. Techniques such as coupled oscillators and nonlinear springs (e.g., magnetic springs) have been introduced to dynamically adjust resonance frequency and broaden operational bandwidth, enhancing energy conversion efficiency. Furthermore, advancements in energy harvesting circuits, particularly the Synchronized Switch Harvesting on Inductor (SSHI) technique, have demonstrated up to 900% improvements in energy yield compared to conventional methods, even under frequency deviations.

Keywords: Energy Harvesting (EH), (WSN), (OFDM), (SSHI), Internet of Things (IoT)

I. INTRODUCTION

Mechanical energy is widely available in environments where Wireless Sensor Networks (WSN) could be deployed, making the conversion of this energy, particularly from ambient vibrations, into electrical power highly attractive for powering these sensors [1-3]. Sources of mechanical energy include human motion or vibrating structures [4-5]. The frequency of vibrations depends on the source: it is generally below 10 Hz for human movement and above 30 Hz for vibrating machines [6-7]. In the context of WSNs, communication often relies on advanced techniques such as Orthogonal Frequency Division Multiplexing (OFDM) [8-9]. This technique is especially suitable due to its robustness against interference and complex propagation environments OFDM systems are essential for ensuring effective communication between sensors, particularly when the harvested mechanical energy is used to power these devices However, designing OFDM circuits in WSNs imposes strict power constraints, as ambient vibrations must provide sufficient energy to sustain transmission [10]. In industrial applications, where vibration levels are often very low (below 1 m/s²) and their frequency corresponds to the power line frequency (50 or 60 Hz, or their harmonics) [11-12], it becomes critical to exploit resonant inertial generators. These devices convert mechanical energy into electrical energy but have an important limitation: their optimal operation is restricted to a single frequency. If the resonance frequency does not match the frequency of ambient vibrations, the power generated decreases significantly, compromising the power supply for OFDM circuits to address these limitations, adaptive kinetic energy harvesting devices have been developed [13-14]. These systems extend the operational frequency range of generators by dynamically adjusting the resonance frequency or increasing the bandwidth. For example, structures with different resonance frequencies, coupled oscillators, or nonlinear springs (such as magnetic springs) can be integrated into devices powering OFDM communication circuits. These mechanisms optimize energy efficiency while ensuring reliable transmission in WSNs, even in environments with low mechanical energy. By combining advanced kinetic energy harvesting techniques with robust OFDM systems, it is possible to design autonomous WSNs that can operate in complex environments, ensuring effective and sustainable communication between sensors [15]. Recent studies primarily focus on the

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optimization of energy-harvesting structures. However, in many cases, the electrical outputs of these devices are insufficient to directly power electronic systems, necessitating efficient methods for accumulating and storing the harvested energy to enable self-powered systems. Research has demonstrated that using capacitors or rechargeable batteries for energy storage in piezoelectric harvesting systems can effectively accumulate energy [16, 17]. Efficient circuit designs for storing harvested charge or delivering it to the load have shown an increase in power output of up to 400% under high excitation levels [18].

In contrast to linear impedance matching techniques [19]., recent advancements introduced the synchronized switch harvesting on inductor (SSHI) technique, significantly improving energy conversion. This method achieves up to 900% higher energy yields compared to traditional methods [20]. Further improvements include extending SSHI for pulsed excitations and proposing circuits with reduced energy dissipation. Recent analysis highlights the SSHI technique's effectiveness [6] and proposing circuits with reduced energy dissipation [21], particularly for systems with moderate electromechanical coupling. It was found to be less sensitive to frequency deviations and less affected by imperfections in voltage inversion compared to conventional approaches, ensuring consistent energy harvesting performance and proposing circuits with reduced energy dissipation [22].

The design of Energy Harvesting Internet of Things (EH-IoT) systems requires careful consideration to ensure that existing electronic components (e.g., microprocessors, sensors, and radios) can be easily integrated into these new devices. While the specific architecture of an EH-IoT device may vary from one vendor to another, the general structure and components are illustrated in Fig 1. In this architecture, the main modification involves replacing traditional batteries with an energy harvesting module, while the other IoT components (sensors, microcontrollers, radios) remain largely unchanged and can be powered by the module. This design simplification is made possible by splitting the energy harvesting system into two main parts: an energy harvesting transducer and a power management module. The power management module typically includes storage elements, DC-DC converters, and rectifiers if the energy source is alternating current (AC).

The transducer plays a crucial role in converting ambient energy available in the environment into usable electrical energy for the IoT system. Different transduction methods and materials are used to harvest energy from various sources such as kinetic energy (motion, vibration, mechanical energy), solar energy, radio frequency (RF) energy, and thermal energy. Each energy source operates under different conditions: for example, kinetic energy harvesting depends on movement or vibration, solar cells require exposure to light, thermal energy can work with temperature differences, and RF energy is captured from ambient radio signals. These energy sources vary in their availability, and choosing the appropriate energy harvesting method is essential for optimizing EH-IoT performance. For instance, it would not be practical to use solar energy harvesting in environments where light is scarce.

However, one of the challenges with these transducers is that the energy they generate is often unstable, meaning additional circuitry is required to regulate the power. This regulation is typically handled by a power management module, which can handle both direct current (DC) and, if necessary, alternating current (AC). The module includes power converters and rectifiers to provide a stable DC output suitable for powering IoT components. Furthermore, because the energy generation is often intermittent, energy storage devices such as capacitors or rechargeable batteries are integrated into the power management system to store the energy for later use.

Section 2 describes the methods and theoretical electromechanical foundations of vibrational energy harvesting systems, and discusses the methods for rectifying the voltage delivered by piezoelectric sensors. Section 3 presents simulations with MATLAB and spice of the two rectification techniques, namely the standard technique and the parallel SSHI technique, in order to provide a comparison between the two techniques. Section 4 provides conclusions.

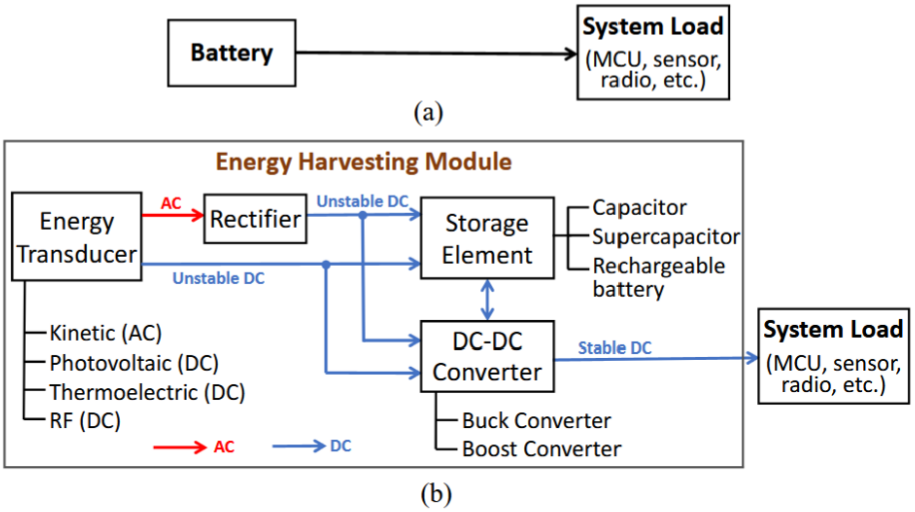

Fig. 1: Generic system architecture for (a) conventional IoT powered by a battery and (b) emerging EH-IoT, where the battery is replaced with an energy harvesting module.

II. MATERIALS AND METHODS

The table 1 provides a detailed analysis of various energy transduction sources, specifying the type of transduction, the associated signal, the operating principle, the activation requirements, and the energy densities generated in specific contexts. This overview highlights the mechanisms by which these technologies exploit different forms of energy (mechanical, electromagnetic, luminous, etc.) to convert physical phenomena such as vibrations, magnetic fields, or light variations into usable energy. The indicated energy density values depend on environmental conditions, such as wind speed, light intensity, or distance from an electromagnetic emission source. This table allows for an evaluation of the potential of each source in practical energy harvesting applications.

Source	Transduction	Signal	Principle	Requirement	Energy Density
	Piezoelectric	AC	compression on crystalline materials	Motions or vibrations	Walking: 49μW/cm2@3km/h,
	Electromagnetic	AC	Changeof magnetic field under movement		piezoelectric EH, knee bending [25]
	Electrostatic	AC	Change of electrical field under movement		Wind: 370μW/cm2@15m/s
Solar	Photovoltaic	DC	Convert light into electricity	Bright environment	Transparent:7mW/cm2@128klux [28] Opaque: 26.7mW/cm2@128klux [29] Opaque : 16μW/cm2@400lux [30] Bright environnement

Table II. Standard AC-DC Conversion System for Energy Harvesting

A. Modeling of the piezoelectric energy harvester.

Consider an energy conversion device consisting of a vibrating piezoelectric generator coupled to an energy storage system. If the resonance modes of the generator are widely separated, the system can be modeled as a mass-spring-damper system with a piezoelectric element, as shown in Fig 2, This model consists of a piezoelectric element attached to a mechanical structure, where a force $F_E(t)$ is applied to a mass M, connected to a spring with stiffness K, a damper with coefficient D, and a piezoelectric component characterized by a coefficient α and a

capacitance C_p. The coefficients, which depend on the material properties and the design of the harvesting devices, can be computed using standard modal analysis

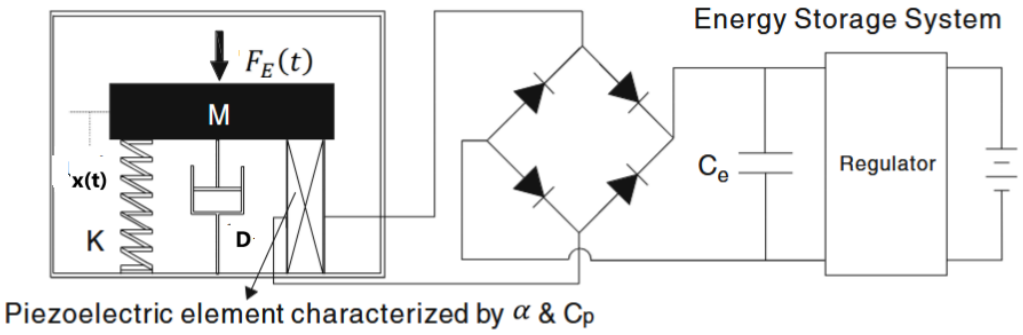

Fig. 2. Configuration of piezoelectric energy harvester

For instance, consider a triple-layer piezoelectric beam mounted cantilevered, polarized in the thickness direction, as shown in Fig 2. The electric field is applied in this direction, and deformation occurs in the axial direction. Thus, the system operates in the transverse mode or mode 3-1. The effective coefficients can be obtained through modal analysis. The expressions for these coefficients are given in (1)(2)(3) and (4) as follows:

$$M = \beta_M\left(m_p + m_b\right) + m_a \tag{1}$$

$$K = \beta_K S\left[\left(\frac{2}{3}\frac{t^3}{L^3} + \frac{ht^2}{L^3} + \frac{1}{2}\frac{th^2}{L^3}\right)C_E p_{11} + \frac{1}{12}\frac{h^3}{L^3}C_E b_{11}\right] \tag{2}$$

$$\alpha = \beta_\alpha \frac{S(h+t)}{2L} e_{31} \tag{3}$$

$$C_p = \frac{SL}{2t}\epsilon_S 33 \tag{4}$$

where β_M, β_K, and β_α are constants obtained from the Rayleigh-Ritz approximation, e_{31} and $\epsilon_S 33$ are the piezoelectric and dielectric constants, respectively, S and L are the beam width and length, and t, h, $C_E p_{11}$, $C_E b_{11}$, m_p, m_b, and m_a are the geometric and material properties of the structure.

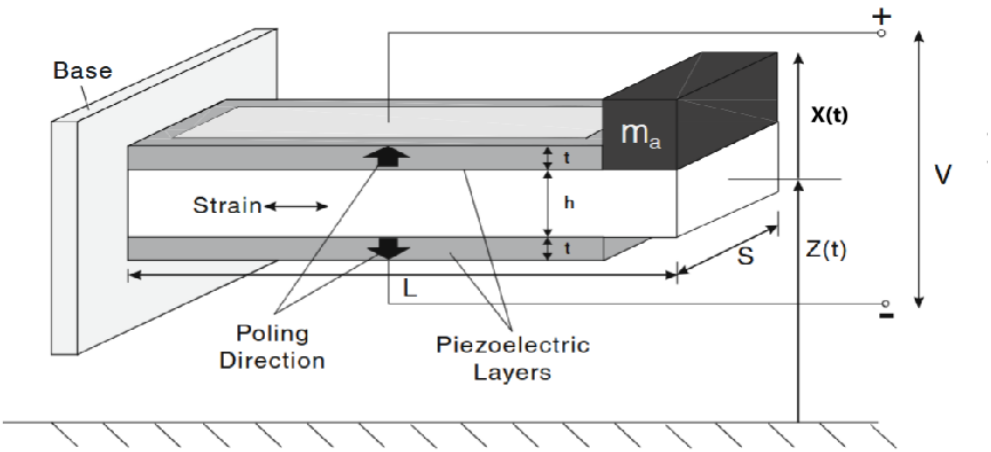

Fig.3. Cantilever Piezoelectric Generator with Three Layers Operating in the {3–1} Mode, Excited by Acceleration ¨x(t).

Fig.3. depicts a cantilever piezoelectric generator, consisting of three layers, operating in the {3–1} mode, where the base is excited by an acceleration ¨x(t). Another piezoelectric generator, operating in the longitudinal mode or 2-3, has been recently developed by Jeon et al. with interdigital electrodes, this mode offers an advantage, as the

longitudinal piezoelectric effect is generally stronger than the transverse effect ($d_{33} > d_{31}$). Let x represent the displacement of the mass M, and V the voltage across the piezoelectric material. The dynamic equations of the piezoelectric generator are given by (5) and (6):

$$M\ddot{x} + D\dot{x} + K_p x + k\dot{x} + \alpha V = F_E(t) \tag{5}$$

$$-k\dot{x} + \alpha\dot{V} = -I(t) \tag{6}$$

where $I(t)$ is the current flowing through the circuit, shown in Fig. 3. Assuming the majority of energy harvesting applications for piezoelectric materials rely on periodic vibrations of the piezoelectric elements, the vibrating generator is assumed to be excited near its resonant frequency in a harmonic manner in (7):

$$F(t) = F_0 \sin(\omega t) \tag{7}$$

. where F_0 is the amplitude of the force and ω is the angular frequency of the vibration. The transfer function of the system between displacement x and force F_p is obtained by taking the Laplace transform given in (8)

$$Ms^2X(s) + (D + k)sX(s) + K_pX(s) = F_E(s) \tag{8}$$

Thus, the transfer function H(s) is given in (9):

$$H(s) = \frac{X(s)}{F_p(s)} = \frac{1}{Ms^2 + (D + k)s + K_p} \times \frac{1}{K_p} \tag{9}$$

Response under Sinusoidal Excitation. For sinusoidal excitation $F_E(t) = F_0 \sin(\omega t)$, the system response is given in (10):

$$X(t) = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} F_0 \sin(\omega t - \varphi) \tag{10}$$

where the phase shift φ is given by: $\varphi = \arctan\left(\frac{2\zeta\omega_n}{\omega_n^2 - \omega^2}\right)$. The instantaneous power $P(t)$ generated by the piezoelectric transducer is: $P(t) = F_p(t) \cdot \dot{X}(t)$ For sinusoidal excitation, this simplifies to (11):

$$P(t) = \frac{m\zeta X_0^2 \left(\frac{\omega}{\omega_n}\right)^3 \omega^3}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2} \tag{11}$$

The maximum power is obtained when $\omega = \omega_n$, giving: $P_{\max} = \frac{mX_0^2\omega_n^3}{4\zeta}$. This shows that the maximum power depends on the mass m, displacement amplitude X_0, natural frequency ω_n, and damping ζ

Modes of Operation The most common modes are: Mode 33: Force and voltage are aligned. Mode 31: Force is applied axially, and voltage is generated perpendicularly. The harvested power depends on mechanical and electrical parameters. The expression for maximum power shows that it can be optimized by adjusting damping ζ, mass m, and frequency ω_n. The natural frequency is crucial for maximizing power, as it defines resonance.

Role of the Natural Frequency ω_n .The natural frequency ω_n determines the resonance of the system and is given by: $\omega_n = \sqrt{\frac{K}{M}}$ where K is stiffness and M is mass. The natural frequency dictates how efficiently the system converts mechanical energy into electrical energy, maximizing harvested power.

B. Electromechanically Model and Power Analysis

In the electromechanical model of the piezoelectric energy harvester, the mass m is represented by an inductance L_m, the damping D by a resistance R_m, and the stiffness K_p by a capacitance C_m. The force applied to the generator is denoted F_E, and the piezoelectric element generates voltage V and current I in response to mechanical vibrations. The coupling factor α links the mechanical deformations to voltage variations. The electrical diagram shown in Fig 4 models the mechanical components using electrical elements. In the case of a mechanical excitation maintained at the resonant frequency,

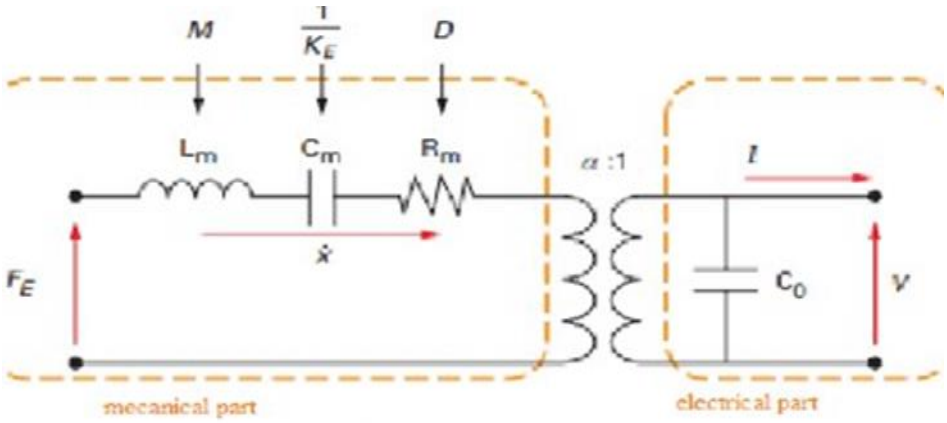

Fig.4. Electrical diagram of the system

The energy generator considered here is connected to a storage circuit, since an electrochemical battery requires a stable DC voltage, while a vibrating piezoelectric element generates an AC voltage, an appropriate circuit is needed to ensure compatibility. Typically, an AC–DC rectifier with a filtering capacitor C_e is used to convert and smooth the output voltage 1). A controller placed between the rectifier output and the battery regulates the resulting voltage. Fig5(a) illustrates a simplified circuit often used in the design analysis of energy harvesting systems. This circuit helps estimate an upper limit for the power that the piezoelectric generator can deliver under a given excitation. In this configuration, the battery and regulator are replaced by an equivalent resistor R, and V_{DC} represents the rectified voltage across it.

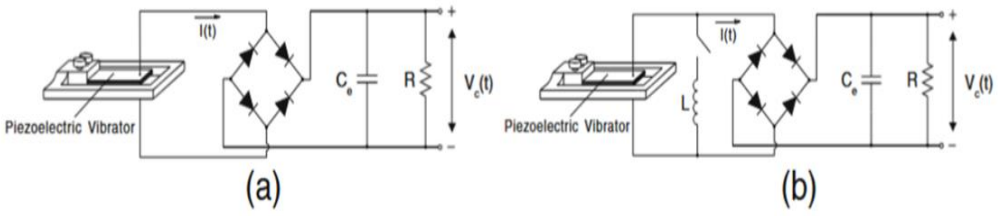

Fig. 5. a Simplified standard energy harvesting circuit based on an AC/DC rectifier's b. SSHI energy harvesting circuit based on an AC/DC rectifier

To achieve a stable DC voltage, it is commonly assumed that the capacitor C_e is large enough to ensure that the rectified voltage V_C remains nearly constant. The voltage $V_C(t)$ can be expressed as the sum of its average value $\langle V_C(t)\rangle$ and a ripple component V_{ripple}. If the time constant RC_e is much greater than the oscillation period of the generator, the average $\langle V_C(t)\rangle$ becomes independent of C_e, while the magnitude of V_{ripple}, which depends on C_e, decreases as C_e increases. In this case, $V_C(t) \approx \langle V_C(t)\rangle$, and V_C can be used for simplicity in notation. The rectifying bridge shown in Fig. 5.a. is assumed to be ideal. When the piezoelectric voltage $|V_p|$ is below V_C, the bridge is blocked, and no current flows. When $|V_p|$ equals V_C, the bridge conducts, and V_p is maintained at V_C. As $|V_p|$ decreases, conduction ceases again. Consequently, the voltage $V_p(t)$ varies proportionally with displacement $x(t)$ when the bridge is blocked or remains equal to V_C when the bridge conducts

1)Standard Interface

The model (5), (6) and (7) are developed at the resonance mode of the device, and therefore, a single-mode vibration of the structure at steady-state operation is expected with given in (12)

$$x(t) = x_0 \sin(\omega t - \alpha) \tag{12}$$

, where x_0 is the magnitude and α is the phase shift. This assumption of choosing the sinusoidal form for displacement has been made by Guyomar et al. (2005) excluding the effect of the phase shift α. Shu & Lien (2006a) have included this effect and validated it both numerically and experimentally for the standard interface. The

corresponding waveforms of $x(t)$ and $V_p(t)$ are shown in Fig. 6(a). Let $T = \frac{2\pi}{\omega}$ be the period of vibration, and t_i and t_f be two-time instants ($t_f - t_i = \frac{T}{2}$) such that the displacement x undergoes from the minimum $-x_0$ to the maximum x_0, as shown in Fig. 5(a). Assume that $\dot{V}_p \geq 0$ during the semi-period from t_i to t_f. It follows that (13)

$$\int_{t_i}^{t_f} \dot{V}_p\,(t)dt = V_C - (-V_C) = 2V_C. \tag{13}$$

Note that $C_e\dot{V}_C(t) + \frac{V_C}{R} = 0$ for $t_i < t < t_f$ during which $|V_p| < V_C$, and $I(t) = C_e\dot{V}_C(t) + \frac{V_C}{R}$ for $t_i \leq t < t_f$ during which the rectifier conducts.This gives the (14)

$$-\int_{t_i}^{t_f} I\,(t)dt = -\frac{T}{2}\frac{V_C}{R} \tag{14}$$

since the average current flowing through the capacitance C_e is zero; i.e., $\int_{t_i}^{t_f} C_e\,\dot{V}_C(t)dt = 0$ at the steady-state operation.The integration of equation (6) from time t_i to t_f is therefore given in (15)

$$-2\alpha x_0 + 2C_pV_C = -\frac{T}{2}\frac{V_C}{R} \tag{15},$$

which simplifies to (16):

$$V_C = \frac{\omega\alpha R}{\omega C_p R + \frac{\pi}{2}}x_0 \tag{16}$$

Thus, from (16), x_0 has to be determined to decide V_C. There are three approaches in the recent literature for estimating x_0. Guyomar et al. (2005), Ottman et al. (2002), and Shu & Lien (2006a) propose estimations based on different assumptions. The first model approximates the piezoelectric device as a current source in parallel with its internal parasitic capacitance C_p (Jeon et al., 2005; Ng & Liao, 2005; Ottman et al., 2002). This approach assumes the internal current source of the generator is independent of the external load impedance. However, the amplitude of the current source is closely related to that of displacement, which depends not only on mechanical damping but also on electrical damping at the resonant vibration (Lesieutre et al., 2004; Shu & Lien, 2006b). This assumption is therefore not suitable when the effect of electrical damping is significant. Guyomar et al. (2005) proposed another estimation that accounts for the electromechanical coupling effect, and their estimation is based on the assumption that the external forcing function and the velocity of the mass are in-phase, neglecting the phase shift α. Instead, Shu & Lien (2006a) included the phase factor in their improved analysis, deriving analytic expressions for the displacement magnitude x_0, rectified voltage V_C, and harvested average power P.The expression for x_0 is given in (17):

$$x_0 = \frac{F_0}{K}\left[\left(2\zeta_m + \frac{2k_e^2 r}{\left(r\zeta + \frac{\pi}{2}\right)^2}\right)^2\zeta^2 + \left(1 - \zeta^2 + \frac{k_e^2 r\zeta}{r\zeta + \frac{\pi}{2}}\right)^2\right]^{\frac{1}{2}} \tag{17}$$

.

The rectified voltage V_{DC} is given by (18):

$$V_C = \frac{F_0}{\alpha}\left[\frac{r\zeta}{r\zeta + \frac{\pi}{2}}\right]k_e^2\left[\left(2\zeta_m + \frac{2k_e^2 r}{\left(r\zeta + \frac{\pi}{2}\right)^2}\right)^2\zeta^2 + \left(1 - \zeta^2 + \frac{k_e^2 r\zeta}{r\zeta + \frac{\pi}{2}}\right)^2\right]^{-\frac{1}{2}} \tag{18}.$$

The harvested power P is given by (19):

$$P = \frac{F_0^2 w_{sc} M}{\left(r\zeta + \frac{\pi}{2}\right)^2}k_e^2\zeta^2 r\left[\left(2\zeta_m + \frac{2k_e^2 r}{\left(r\zeta + \frac{\pi}{2}\right)^2}\right)^2\zeta^2 + \left(1 - \zeta^2 + \frac{k_e^2 r\zeta}{r\zeta + \frac{\pi}{2}}\right)^2\right] \tag{19}$$

The dimensionless variables are defined as:

$$k_e^2 = \frac{\alpha^2 K}{C_p}, \zeta_m = \frac{D}{2\sqrt{KM}}, w_{sc} = \sqrt{\frac{K}{M}}, \zeta = \frac{\omega}{w_{sc}}, r = \frac{C_p w_{sc}}{R}.$$

Here, k_e^2 is the electromechanical coupling factor, ζ_m is the mechanical damping ratio, w_{sc} is the natural frequency of the system under short-circuit conditions, ζ is the normalized frequency ratio, and r is the normalized resistance. The two resonances for the system are defined as: $\zeta_{sc} = 1, \zeta_{oc} = \sqrt{1 + k_e^2}$.Where ζ_{sc} is the short-circuit frequency ratio and ζ_{oc} is the open-circuit frequency ratio. The shift in the device natural frequency is pronounced when the coupling factor k_e^2 is large.

2) SSHI-Harvesting Circuit

An SSHI electronic interface consists of adding a switch and an inductance L connected in series and is in parallel with the piezoelectric element, as shown in Fig. 5(b). The electronic switch is triggered according to the maximum and minimum of the displacement of the mass, causing the processing of piezoelectric voltage to be synchronized with the extreme values of displacement. To illustrate the electrical behavior of this nonlinear processing circuit, consider the harmonic excitation given by (7). In view of the single-mode excitation, the mechanical displacement $x(t)$ is assumed to be sinusoidal as in (12) in steady-state operation. The validation of this assumption has been examined by considering the output voltage (Shu et al. 2007). The waveform of the piezoelectric voltage $V_p(t)$, however, may not be sinusoidal and is dependent on the specific type of the interface circuit connected to the piezoelectric element. To see it, let $T = \frac{2\pi}{\omega}$ be the period of mechanical excitation and t_i and t_f be two-time instants such that the displacement $u(t)$ undergoes from the minimum $-u_0$ to the maximum u_0, as illustrated in Fig. 6.b. The switch is turned off most of the time during this semi-period (t_{+i}, t_f). When it is turned on at the time instant t_i, $|V_p(t)|$ remains lower than the 90 rectified voltage V_c. So, the rectifying bridge is open circuited, and an oscillating electrical circuit composed by the inductance L and the piezoelectric capacitance C_p is established, giving rise to an inversion process for the piezoelectric voltage V_p. Specifically, let Δt be the half electric period of this oscillating $L - C_p$ circuit. It is equal to $\pi\sqrt{LC_p}$.We assume that the inversion process is quasi-instantaneous in the sense that the inversion time is chosen to be much smaller than the period of mechanical vibration; i.e., $\Delta t = t_{+i} - t_i < T$. The switch is kept closed during this small time period Δt, resulting in the reverse of voltage on the piezoelectric element; i.e., $V_p(t_{+i}) = -V_p(t_i)e^{-\pi/2Q_I}$, $q_I = e^{-\pi/2Q_I}$, as shown in Fig. 6(b). Here, Q_I is the inversion quality factor due to the energy loss mainly from the inductor in series with the switch. As a result, the current outgoing from the piezoelectric element through the rectifier during a half vibration period can be obtained by integrating Equation (6) from time t_{+i} to t_f given in (20)

$$\int_{t_f}^{t_{+i}} -\alpha\dot{x}(t) + C_p\dot{V}_p(t)dt = -2\alpha x_0 + C_p\left(1 - e^{-\frac{\pi}{2Q_I}}\right)V_c = -\frac{\frac{T}{2}V_c}{R} \tag{20}$$

since the rectifier bridge is blocking during the inversion process and the inversion time $\Delta t < T$. The relation between the magnitude of displacement x_0 and the rectified voltage V_c is therefore obtained by (21)

$$V_c = 2R\alpha\omega(1 - q_I)C_p\omega + \pi x_0. \tag{21}$$

The rest of the derivation is to estimate the magnitude of displacement u_0 and the phase shift θ, and we refer to the work by Shu et al. (2007) for details. The results for normalized displacement magnitude u_{SSHI0}, rectified voltage V_{SSHIc}, and average-harvested power P_{SSHI} are given, respectively, by (22) (23) and (24)

$$x_{SSHI0} = \frac{x_{SSHI0}}{\frac{F_0}{K}} = \frac{1}{\left\{2\zeta_m + \frac{\left(2\left[1 + \frac{r\omega}{2\pi(1 - q_I^2)}\right]\right)k_e^2 r}{\left(\frac{(1 - q_I)}{2r\omega} + \frac{\pi}{2}\right)^2}\right\}\omega} \tag{22}$$

$$V_{SSHIc} = \frac{V_{SSHI}}{\frac{F_0}{\alpha}} = \frac{rk_e^2}{\left(\frac{(1-q_I)}{2r\omega} + \frac{\pi}{2}\right)\left\{2\zeta_m + \frac{\left(2\left[1 + \frac{r\omega}{2\pi(1-q_I^2)}\right]\right)k_e^2 r}{\left(\frac{(1-q_I)}{2r\omega} + \frac{\pi}{2}\right)^2}\right\}} \tag{23}$$

$$P_{SSHI} = \frac{P_{SSHI}}{\frac{F_0^2}{w_{sc}M}} = \frac{k_e^2 r}{\left(\frac{(1-q_I)}{2r\omega} + \frac{\pi}{2}\right)^2\left(\{(2\zeta_m + \frac{\left(2\left[1 + \frac{r\omega}{2\pi(1-q_I^2)}\right]\right)k_e^2 r}{\left(\frac{(1-q_I)}{2r\omega} + \frac{\pi}{2}\right)^2}\}\right)^2} \tag{24}$$

where all are expressed in terms of dimensionless parameters defined by

$$k_e^2 = \frac{\alpha^2 K}{C_p}, \zeta_m = \frac{D}{2\sqrt{KM}}, w_{sc} = \sqrt{\frac{K}{M}}, \zeta = \frac{\omega}{w_{sc}}, r = \frac{C_p w_{sc}}{R}.$$

Finally, Guy Omar et al. (2005) have used the in-phase assumption to analyze the electrical performance of the power generator using the SSHI interface. To be precise, they have assumed that the external forcing function and the velocity of the mass are in-phase, giving rise to no phase shift effect in their formulation. The following summaries their results for comparisons:

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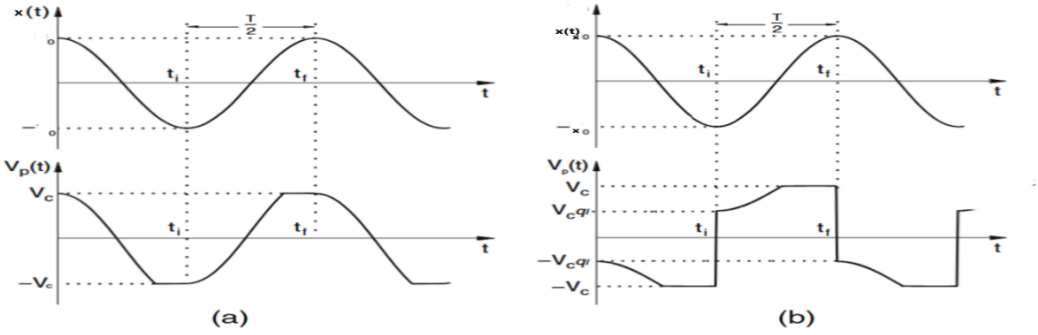

Fig.6. Typical waveforms of displacement and piezoelectric voltage for (a) the standard and (b) the SSHI electronic interfaces

III. SIMULATION

This simulation presents a standard energy recovery system, illustrated in Fig 7, along with the corresponding waveforms under typical operating conditions. The system is characterized by a current, a capacitance, a resistance, and an operating frequency. Under these conditions, the direct current (DC) power dissipated in the system is given. The Fig highlights how the system recovers energy from mechanical vibrations, showing the corresponding variations in voltage and current. These waveforms are essential to understanding the system's efficiency in converting mechanical energy into electrical power, providing valuable insights into the energy recovery process. Fig 8, on the other hand, presents the diagram of the nonlinear optimization for the series SSHI technique, based on Allspice. This circuit integrates a piezoelectric generator composed of a current source I_P in parallel with a capacitance C_P, representing the interaction between the deformation of the piezoelectric material and the electrical circuit. The generated signal is then processed by a series switch consisting of two transistors, a P-MOS (M_1) and an NMOS (M_2), which regulate the energy flow. This circuit is coupled with an inductive coil L_1 of 14 mH, likely serving as energy storage or current spike filtering. A control circuit, including an electronic differentiator, adjusts the signal before it reaches the comparator, which detects the zero-crossing points of the current, corresponding to the extrema of the displacement of the piezoelectric generator, indicating the points of maximum force or maximum vibration of the system. Finally, Fig 9 presents the waveform signals for the parallel SSHI technique, detailing key parameters such as resistance, inductance, capacitance, and the generated power. These waveforms provide an overview of the system's efficiency and dynamics under specific operating conditions. Fig 7 illustrates the standard energy recovery system along with the corresponding waveforms under

The diagram of the nonlinear optimization for the serial SSHI technique, is presented in Fig 8 based on Allspice. The described circuit integrates a piezoelectric generator composed of a current source I_P in parallel with a capacitance C_P, representing the interaction between the deformation of the piezoelectric material and the electrical circuit. The signal generated by this device is then processed by a series switch formed by two transistors: a P-MOS (M_1) and an NMOS (M_2), which regulate the energy flow. This circuit is coupled to an inductive coil L_1 of 14 mH, likely serving as energy storage or filtering current spikes. The control circuit includes an electronic differentiator consisting of resistors R_2, R_3, and capacitor C_2, forming a network that allows differentiating the piezoelectric signal, extracting rapid variations of the current. This differentiator adjusts the signal before it reaches the comparator, based on an operational amplifier. The comparator detects the zero-crossing of the current, corresponding to the extrema of the displacement X of the piezoelectric generator, indicating the points of maximum force (F_E) or maximum vibration of the system. Finally, the circuit includes a MOSFET-based rectifier, allowing the conversion of the alternating current generated by the piezoelectric generator. Fig. 9gives wave signals of the Parallel SSHI technique, illustrating the synchronization and amplitude characteristics of the signal processing method

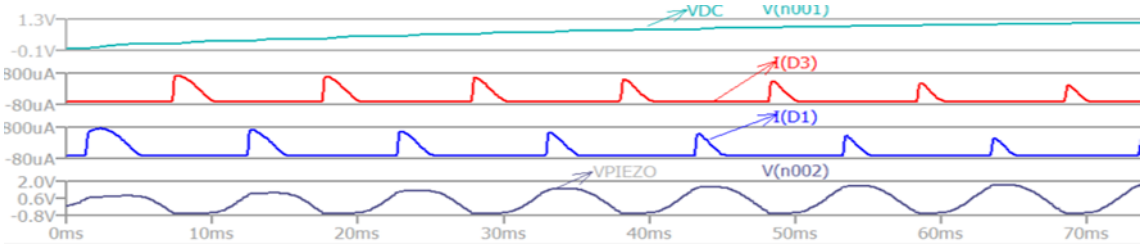

Fig.7. -Standard energy recovery and wave forms for standard technical Ip= 800µA c_0=470nF R=100kΩ f=20HZ PDC= 16.9µw

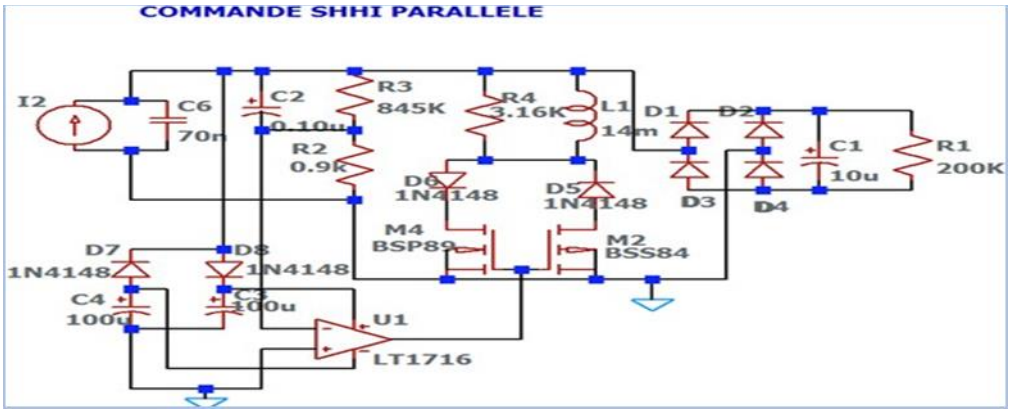

Fig.8. Diagram of parallel SSHI technic

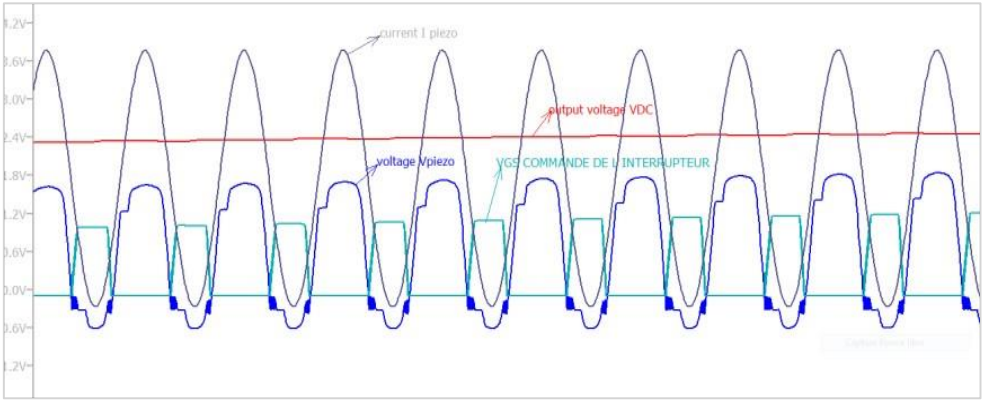

Fig.9. Wave Signals of Parallel SSHI Technique Ip= 800µA c_0=470nF R=100kΩ f=20HZ PDC= 135.2µw

A. Performance of Piezoelectric Energy Harvesting Systems Using standard and SSHI Interface

1) Standard interface

The improved estimates given in equations (17), (18), and (19) for the standard AC-DC interface have been shown to agree well with experimental observations and numerical simulations of equations (5), (6), and (7) under the conditions defined in (7) (Shu & Lien, 2006a). Therefore, these estimates are suitable for evaluating the electrical performance of piezoelectric energy harvesting systems with a standard electronic interface. Specifically, according to equation (19), the harvested average power increases significantly for smaller mechanical damping ratio ζ_m or larger electromechanical coupling coefficient k_e^2, which is consistent with the findings by Badel et al. (2006a). They observed that the power generated using a single crystal was much higher than that generated by a ceramic, due to the fact that the coupling factor k_e^2 of the former was 20 times larger. However, caution is needed because the harvested power tends to saturate for much larger k_e^2, as shown in Fig 10.Normalized power P versus normalized frequency ξ and electromechanical coupling factor k_e^2 under optimal conditions such that $P_{opt}(\xi, k_e^2, \zeta_m) = P\left(r_{opt}(\xi), \xi, k_e^2, \zeta_m\right)$. Here, $r_{opt}(\xi)$ is determined by solving $\frac{\partial P}{\partial r} = 0$, with $\zeta_m = 0.04$. Note that for large k_e^2, there are two identical power peaks at the frequency ratio close to $\xi_{sc} = 1$ and $\xi_{oc} = \sqrt{1 + k_e^2}$. These peaks saturate for much higher coupling factors $k_e^2 \gg 1$.Normalized power P versus normalized frequency ξ and electromechanical coupling factor k_e^2 under optimal conditions such that $P_{opt}(\xi, k_e^2, \zeta_m) = P\left(r_{opt}(\xi), \xi, k_e^2, \zeta_m\right)$. Here, $r_{opt}(\xi)$ is determined by solving $\frac{\partial P}{\partial r} = 0$, with $\zeta_m = 0.04$. Note that for large k_e^2, there are two identical power peaks at the frequency ratio close to $\xi_{sc} = 1$ and $\xi_{oc} = \sqrt{1 + k_e^2}$. These peaks saturate for much higher coupling factors $k_e^2 \gg 1$.The improved estimates in equations (17), (18), and (19) were also compared with uncoupled and in-phase estimates according to the relative magnitudes of the electromechanical coupling coefficient and the mechanical damping ratio. The results from Shu & Lien (2006a) show that the conventional uncoupled solutions and in-phase estimates are valid when the ratio $k_e^2\zeta_m$ is small, but the discrepancies between these different approaches become significant as this ratio increases. In cases where the shift in natural frequency is large and the system's mechanical damping ratio is small, i.e., $k_e^2\zeta_m \gg 1$, the harvested power shows two optima evaluated at $\left(r_{opt_1}, \xi_{opt_1}\right)$ and $\left(r_{opt_2}, \xi_{opt_2}\right)$, where ξ_{opt_1} is close to ξ_{sc} and r_{opt_1} is very small, while ξ_{opt_2} is close to ξ_{oc} and r_{opt_2} is large.

2) Optimal Conditions

Table 2 summarizes the relationship between the system parameters k_e^2 and ζ_m, and the normalized load, displacement, voltage, and power under the optimal resonance conditions for the short-circuit ξ_{sc} and open-circuit ξ_{oc}.Relation between the system parameters k_e^2 and ζ_m and the normalized electric resistance, displacement, voltage, and power at the short-circuit (ξ_{sc}) and open-circuit (ξ_{oc}) resonances. The first optimal pair is designed at the short-circuit resonance ξ_{sc}, with the optimal load $r_{opt_{sc}} \propto \frac{1}{k_e^2\zeta_m}$, while the second optimal pair is designed at the open-circuit resonance ξ_{oc}, with the optimal load $r_{opt_{oc}} \propto \frac{1}{(1+k_e^2)\zeta_m}$. These two conditions yield the same maximum harvested power, which depends solely on the mechanical damping ratio ζ_m. Unlike the harvested power, the displacement is higher at ξ_{sc} than at ξ_{oc}, while the voltage at the first peak is one order of magnitude smaller than that at the second peak.

Normalized harvested power versus normalized electric resistance and frequency ratio. (a) A strongly coupled electromechanical system using the standard AC/DC electronic interface ($k_e^2 = 1.0$, $\zeta_m = 0.04$, $k_e^2\zeta_m = 25$). (b) A weakly coupled electromechanical system using the ideal SSHI electronic interface ($k_e^2 = 0.01$, $\zeta_m = 0.04$, $k_e^2\zeta_m = 0.25$, $Q_I = \infty$). Both systems exhibit identical power peaks at different conditions.

3) Discussions

If the vibration source results from the periodic excitation of some base, the force can be expressed as $F_0 = MA$, where A is the magnitude of the acceleration of the exciting base. From equation (19), the harvested average power per unit mass becomes:

$$\frac{P}{M} = A^2 w_{sc} P(r, \xi, k_e^2, \zeta_m)$$

Since equation (19) is expressed in terms of dimensionless parameters, it provides a normalized power scheme that can be used to compare power harvesting devices of various sizes and vibration inputs to estimate efficiencies.

Conceptually, this formula provides a design guideline to optimize AC-DC power output either by adjusting the electric resistance, selecting suitable operation points, or by adjusting the coupling coefficient through careful structural design. However, there are several challenges to making this approach feasible:

1. Parameter adjustment: It is not always easy to adjust one parameter while keeping others fixed. For example, adjusting the device dimensions can result in simultaneous changes in the dimensionless parameters r, ξ, k_e^2, and ζ_m.
1. Frequency constraint: The natural frequency of the device needs to be below 300 Hz to match the frequency range of common ambient sources. Additionally, the most common geometry for piezoelectric generators is the cantilever beam, whose resonance frequency is inversely proportional to its length, leading to a significant increase in natural frequency at the microscale.
2. $3-3$ actuation mode: It is advantageous to operate in the $3-3$ mode, as it allows better coupling between the mechanical and electrical domains. However, the use of interdigitated electrodes leads to variable performance depending on the distance between the electrodes and the volume of the generator.
3. Increasing coupling coefficient: One method to increase k_e^2 is to apply destabilizing axial loads, as shown in Lesieutre & Davis (1997). However, this also increases device damping, requiring a trade-off between coupling and damping.
4. Resonance frequency: Vibration-based power generators achieve maximum power when their resonance frequency matches the driving frequency. The harvested power decreases significantly if the frequency deviation exceeds 5

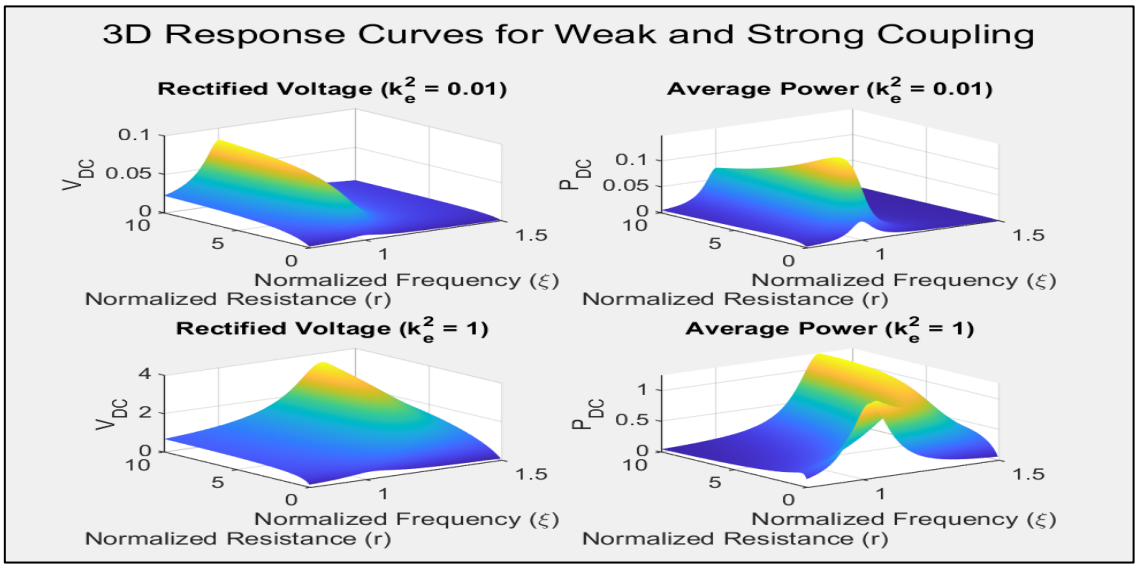

Fig.10. 3D Representation of Normalized Voltage and Power as Functions of Frequency and Resistance for Different Electromechanical Coupling Coefficients

B. **SSHI Interface**

On the other hand, the improved estimations presented in (22), (23 and (24) for the SSHI interface, which incorporate this frequency dependence, have been numerically validated. These revised models thus provide a solid foundation for evaluating the electrical performance of piezoelectric energy harvesting systems integrated with an SSHI interface circuit. Let us consider an ideal case where the piezoelectric voltage inversion V_p is perfectly executed, implying $Q_I = \infty$. this leads to $q_I = 1$, and the normalized harvested power, as defined by equation (22), becomes:

$$P_{\text{SSHI}} = \frac{4}{\pi^2} r k_e^2 \zeta^2 \left\{ 4\left(\zeta_m + \frac{4k_e^2 r}{\pi^2}\right)^2 \zeta^2 + (1-\zeta^2)^2 \right\}$$

It is then possible to determine the optimal load resistance and the normalized power operating at the short-circuit resonance ζ_{sc}, which are given by the following expressions:

$$r_{\text{opt}} = \frac{\pi^2}{4}\frac{1}{k_e^2 \zeta_m}, P_{\text{SSHI}\vee r=r_{\text{opt}},\zeta=1} = \frac{1}{16\zeta_m}$$

From these equations, it is clear that the optimal load resistance is inversely proportional to the ratio $\frac{k_e^2}{\zeta_m}$, while the maximum power depends solely on the mechanical damping coefficient ζ_m and is independent of the electromechanical coupling coefficient k_e^2.When comparing these characteristics with those presented in Table 2 it is evident that the behavior of the energy harvesting system using the SSHI interface resembles that of a highly coupled electromechanical system operating at short-circuit resonance, according to Table 2, another power peak exists, but it is now located at open-circuit resonance. However, the results show that this second power peak moves to infinity within the (r, ζ) space, meaning that only one power peak exists for the SSHI interface, regardless of whether the real electromechanical system is weakly or strongly coupled. Table 2 gives relation between the system parameters k_e^2 and ζ_m and the normalized electrical resistance, displacement, voltage, and power operated at the short-circuit (ε_{sc}) and open-circuit (ε_{oc}) resonances (Shu & Lien 2006a). Note that the condition $k_e^2 \zeta_m \gg 1$ is implied in the analysis

Optimal conditions	Short-circuit (ε_{sc})	Open-circuit (ε_{oc})
Resistance r_{opt}	$\propto \frac{1}{k_e^2 \zeta_m}$	$\propto \frac{1}{(1+k_e^2) k_e^2 \zeta_m}$
Displacement u_{opt}^0	$\propto \frac{1}{\zeta_m}$	$\propto \frac{1}{\zeta_m \sqrt{1+k_e^2}}$
Voltage V_{opt}^c	$\propto \frac{1}{k_e^2 \zeta_m}$	$\propto \frac{1}{\sqrt{1+k_e^2} k_e^2 \zeta_m}$
Power P_{opt}	$\propto \frac{1}{\zeta_m}$	$\propto \frac{1}{\zeta_m}$

Table III.0ptimal Conditions for System Parameters at Short-Circuit and Open-Circuit Resonances

For example, with $k_e^2 = 0.01$ and $\zeta_m = 0.04$, the electromechanical generator is weakly coupled ($\frac{k_e^2}{\zeta_m} = 0.25$). In this case, the power harvested by the standard circuit is relatively low, as illustrated by the following relation:

$$P\left(r_{\text{opt}} = \frac{\pi}{2}, \zeta = 1, k_e^2, \zeta_m\right) \approx \left(\frac{2}{\pi}\frac{k_e^2}{\zeta_m}\right)\frac{1}{16\zeta_m} \ll \frac{1}{16\zeta_m} = P_{\text{SSHI}_{\max}} \text{if} \frac{k_e^2}{\zeta_m} \ll 1$$

However, the introduction of the SSHI circuit leads to a substantial increase in harvested power, which reaches the maximum value of a highly coupled electromechanical generator connected to the standard interface, as shown in Fig 11 (with $k_e^2 = 1.0$, $\zeta_m = 0.04$, and $\frac{k_e^2}{\zeta_m} = 25$). Thus, harvested power increases significantly in the case of any low-coupling SSHI system, but this requires the use of a much larger optimal electrical load, which is proportional to $\frac{1}{k_e^2 \zeta_m}$ .Finally, as often occurs in practical situations, the inversion of the piezoelectric voltage V_p is not perfect ($Q_I = \infty$), leading to a degradation in the performance of the system using the SSHI electronic interface. To study this effect, we take $Q_I = 2.6$ to compare the electrical performance of a piezoelectric vibration-based generator using standard and SSHI electronic interfaces for different $\frac{k_e^2}{\zeta_m}$ ratios. The phase estimates in the equations provided by Guy Omar et al. (2005) lack frequency dependence. Therefore, they cannot predict the system behavior when the driving frequency deviates from the resonance frequency of the system. Since the power reduction is significant due to the frequency mismatch, such an effect cannot be ignored in a practical design. On the other hand, the enhanced estimates for the SSHI interface show frequency dependence and have also been numerically validated by Shu et al. (2007). Consequently, these estimates are suitable for evaluating the electrical performance of the energy harvesting system integrated with an SSHI interface circuit. To see how the SSHI interface improves power extraction, consider an ideal case where the piezoelectric voltage inversion V_p is complete, i.e., $Q_I = \infty$. In this case, the normalized harvested power becomes proportional to ζ_m, the optimal

electrical load resistance, and the normalized power operated at an optimal load is defined in terms of k_e^2 and ζ_m .The comparison of all these characteristics shows that the behavior of the energy harvesting system using the SSHI interface is similar to that of a strongly coupled electromechanical system using the standard interface and operating at resonance in a short-circuit condition ε_{sc}. Moreover, there is another identical power peak operating at the open-circuit resonance, but it is shifted infinitely in the (r, ε) space, and consequently, there is only one power peak for the SSHI electronic interface. The harvested power obtained using the standard harvesting circuit is quite low in this case, whereas the inclusion of the SSHI circuit increases the average harvested power, whose maximum is the same as that of a strongly coupled electromechanical generator connected to the standard interface. Therefore, the harvested power increases significantly for any SSHI system with weak coupling at the cost of using a much larger optimal electrical load, which is proportional to $\frac{1}{k_e^2 \zeta_m}$.As in many practical situations, the inversion of the piezoelectric voltage V_p is not perfect ($Q_I = \infty$), which explains some degradation in the performance of the SSHI electronic interface. The results show that the optimal power obtained using the SSHI interface is three times higher than that obtained with the standard interface. However, there is a significant degradation in performance in this case. Next, if the electromechanical coupling is in the medium range, the performance shows that the harvested power evaluated around the optimal load is less sensitive to frequency mismatches compared to the resonance vibration. Fig10 gives Comparison of harvested power P_{SSHI} for strongly and weakly coupled systems. Finally, for a strongly coupled electromechanical system, the results show that the harvested power presents two identical optimal peaks in the case of the standard interface. In contrast, there is only one power peak in the SSHI circuit, as explained earlier. Unlike the standard case, the peaks of the average harvested power decrease significantly as the load resistances increase, thereby contributing to the increase in energy efficiency.

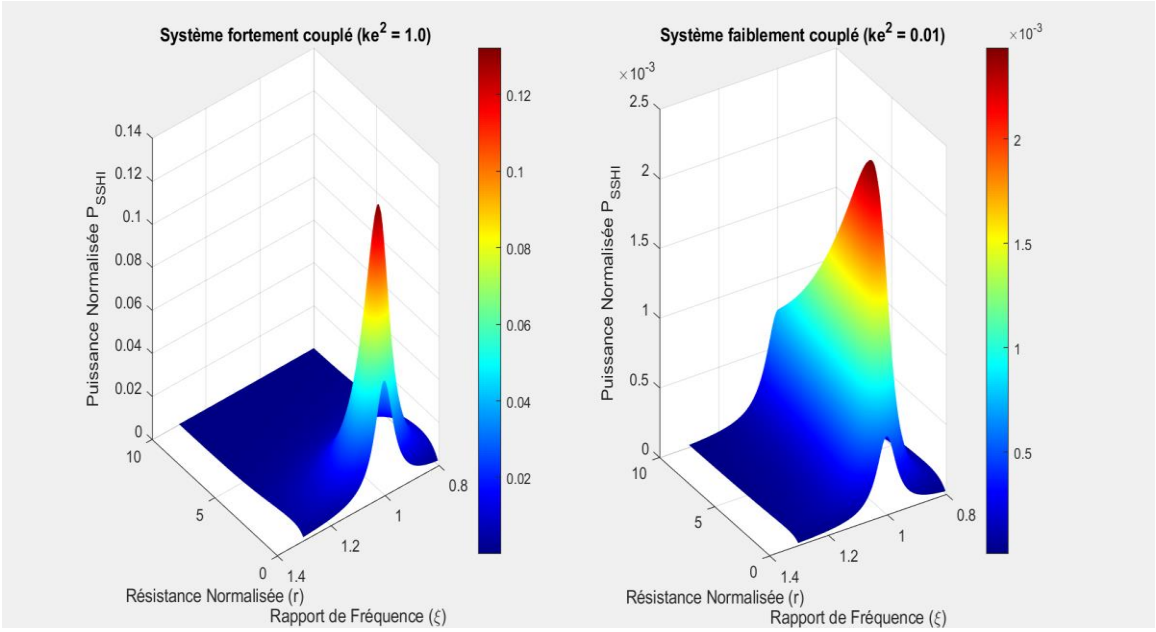

Fig11. Comparison of harvested power P_{SSHI} for strongly and weakly coupled systems.

1. Weakly coupled system ($k_e^2 \ll \zeta_m$): The harvested power with the SSHI interface increases significantly compared to the standard circuit, even with imperfect voltage inversion.

2. Strongly coupled system ($k_e^2 \gg \zeta_m$): The harvested power is reduced as the load resistance increases. However, the SSHI circuit shows a more stable and robust response to frequency variations, providing better energy management.

IV. CONCLUSION

This article presents a theory of energy harvesting based on piezoelectric vibrations, building on the works of Shu & Lien (2006a), Shu & Lien (2006b), and Shu et al. (2007). The theory predicts the electrical behavior of piezoelectric energy harvesting systems, using either the standard or SSHI electronic interface. It demonstrates that the efficiency of energy extraction depends on several factors, such as the characteristics of the input vibrations (frequency and acceleration), the mass of the generator, the electrical load, the natural frequency, the mechanical damping ratio, the electromechanical coupling coefficient of the system, and/or the quality factor of inversion in an SSHI circuit. An expression for the average harvested power, incorporating all these factors, is provided analytically by equations (19) or (24) for the standard or SSHI interfaces. Since the formula is expressed in terms of dimensionless parameters, an effective power normalization scheme is proposed, which can be used to compare the efficiency of energy harvesting devices of various sizes and under different vibration conditions. It is also highly recommended to include these parameters in future publications to facilitate the relative comparison of different devices. Finally, the developed theory points to opportunities for new devices and improvements in existing ones, particularly by showing that optimization criteria vary according to the relative strength of coupling, and that the use of the SSHI technique can broaden the bandwidth of the harvester.

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