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# Hybrid Watermarking Technique for Securing Medical (X-ray) Images Using Slantlet Transform (SLT), Finite Ridgelet Transform (FRT), and Arnold Transform

**Abstract-** This paper proposes a hybrid watermarking technique as a method of enhancing medical X-ray images using the Slantlet Transform, Finite Ridgelet Transform, and Arnold Transform. In this mechanism, the watermark is embedded in X-ray images such that it is quite immune to attacks and causes nothing but minimal loss in image quality. Application of Slantlet Transform (SLT) is preferred because SLT represents the image in compact form with efficient decipherability while embarking on effective watermark embedding. Furthermore, the Finite Ridgelet Transform offers a directionality that enables geometrically invariant watermarks. To offer even higher security as well as a more imperceptible state from geometric alterations or other similar manipulations, Arnold Transform is utilized in scrambling that prior to embedding it in an image sending process where such characteristics of the seal, spatial in nature, get dispersed. Exploratory results show that this hybrid not only preserves the original image but also provides significant robustness against many attackers like noise, cropping, or compression. As such, therefore, it is most appropriate when it comes into play concerning the preservation of medical images in any form of secret communication and storage systems.

**Keywords:** Watermarking; Slantlet Transform (SLT); Finite Ridgelet Transform (FRT); Arnold Transform

## I. Introduction

Medical imaging has gone through digitization, which has made it possible to provide fast diagnoses and store and share patient information easily. X-ray pictures are particularly important for diagnosing and treating different health conditions. As such, they need to be kept safe and private. Nonetheless, with increasing transmission over networks and storing them as part of electronic patient files, unauthorized access, alteration, and other threats can emerge on these pictures [13]. The protection from unauthorized intrusions, retention of patient privacy rights, compliance with the law, and preservation of medical diagnosis accuracy depend on these images being secured and authentic [23]. In this regard, digital watermarking stands out as one major approach aimed at addressing such concerns by introducing some imperceptible information into the image in order to enhance reliability detection of any unauthorized alterations [28].

Digital watermarking is a powerful technique utilized to embed secondary information, commonly referred to as a watermark, into digital images in such a way that it remains unnoticed during normal viewing conditions. The watermark is expected to be so resilient as to survive various picture processing activities, including, for instance, compression, cropping, and noise addition, which are usually part of medical imaging operations. At the same time, however, it should not compromise the quality of the original image since any degradation in picture quality may harm diagnoses made on the basis of them. These factors render watermarking in the area of medicine difficult, involving complex techniques whose aim is to establish a balance between robustness and imperceptibility [15].

A hybrid watermarking technique that employs distinct transformation methods can greatly enhance both security and strength of the entire watermarked process [37]. This paper suggests a hybridized watermarking

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strategy for safeguarding medical X-ray images via inclusion into the Slantlet Transform (SLT) and FRT together with the Arnold Transform. The advantages provided by combining these three approaches render this scheme particularly right for the security of such a sensitive area as protection against unauthorized access to images taken during radiological examination.

Generalizations of the Discrete Wavelet Transform (DWT) are found in SLT that possesses shorter support length, enabling better time-frequency localization [21]. Because of its features, it acts as an effective tool for watermark embedding, as this allows for every area of the image that has less perceptible standpoint to embed a watermark, thus minimizing any visual distortions present [25][31]. Moreover, the multi-resolution analysis feature ensures that watermarks are distributed across various frequency bands, strengthening their resilience against distinct attacks from other changes and disturbances [32].

Used as a second strong tool within the proposed hybrid technique is FRT. In particular, it captures directional images in ways particularly applicable for medical images like X-ray photographs, which often demonstrate clearly defined linear structures [20]. By putting the watermark into the ridgelet domain, such an approach can utilize the directional sensitivity of FRT, making the watermark resistant to common geometrical transformations in image processing such as rotation or scaling [24]. Additionally, the ridgelet domain is less susceptible to certain types of noise that lead to further robustness for this watermark.

The proposed method utilizes the Arnold Transform to scramble the watermark before embedding it into the image for further amplification of security levels over those already in existence. The Arnold Transform is a strong chaotic mapping that actually scrambles the watermark spatial coordinates, so it becomes impossible for people without knowledge of the scrambling parameters to detect or to retrieve the watermark itself 19. This scrambling process ensures that even if an attacker successfully gets hold of the watermarked image, he/she would struggle to retrieve the information inside unless he/she has the appropriate decryption key. Thus, through this process of Arnold Transform, the confidentiality of the image is enhanced through watermarking, whereas SLT and FRT ensure robustness. The integration of these three transforms in one hybrid system results in a remarkable solution for the protection of medical X-ray images against unauthorized access.

The proposed combination of methods, which are the Slantlet Transform, Finite Ridgelet Transform, and Arnold Transform, has presented a strong solution geared towards securing medical X-ray pictures. By exploiting each of the methods offered here above, this new technique achieves a high degree of imperceptibility as well as strength while ensuring that any picture operations which might be performed cannot delete or damage watermark integrity. This hybrid approach not only prevents the degradation of medical images but also authenticates them; therefore, it is enhancing the security level of digital health information in general.

## II. RELATED WORK

Need for confidentiality, authentication, and integrity of medical data over the last few years that has driven the development of significant advancements in digital watermarking, especially for medical images. For example, most diagnostic procedures require medical images such as X-rays that must not be accessed or altered without authorization. This resulted in research on various watermarking techniques particularly hybrid methods where security and robustness are achieved through a combination of several transforms.

One of the basic methods in digital watermarking is embedding information directly into the image's spatial domain, e.g., Least Significant Bit (LSB) [18]. Although these are easier and less computationally intensive than other methods, they are highly susceptible to various types of image processing, such as compression, noise addition, and filtering. For example, Cox et al. (1997) [5] introduced one of the first spatial domain watermarking techniques, which, although effective, was prone to attacks such as JPEG compression or geometric distortions.

In order to tackle this issue, many scholars are looking for transform domain techniques that provide better robustness by embedding the watermark into frequency components of the image. Among them, DWT is probably one of the most prevalent techniques used [16]. DWT-based watermarking methods break an image into subbands, enabling low-frequency components to accommodate a new watermark, thus achieving a tradeoff between robustness and invisibility [17][22]. In their work, Barni et al. (2001) [6] showed that watermarks embedded within wavelet domains are more resistant than any other method to ordinary assaults on medical image processing. Nonetheless, its directional insensitivity limits DWT's effectiveness when dealing with images containing significant linear features like x-ray pictures.

Researchers have looked into taking plain images to shore up against their directional limitations. Strategies combining DWT and DCT have been studied, which gain from the advantages of both transformations. For instance, Kundur & Hatzinakos (2004) proposed a method that uses DWT as a preprocessor for the Claude Shannon type watermarks within it through two schemes; one was filtering out noise to improve clarity while keeping some of every other pixel so that no more than half whites remain anywhere around or close to an output equal to x2. Similarly, SVD-based techniques jointly with DWT have been suggested as a means of improving robustness. Notwithstanding, such methods still struggle with images showing complex directional features even if they exceed simple spatial or transformation domain techniques.

One way to make the best out of these ambivalent transformations is by using the Slantlet transform (SLT), which is an improved version of DWT, shorter and having better time-frequency localization. Due to the several excellent features, among them strength and invisibility, SLT has been often used in various watermarking algorithms. According to Yang et al. (2010) [8], during their research on applying SLT for watermarking of medical images, it has been found that such a tool presents a better image quality as well as enhanced attack resistance. This implies that SLT produces several levels of resolution such that the watermark extends to cross several frequency bands, thus better resisting distortions such as noise or compression. Except in SLT, FRT is known for its ability to capture directional information that exists in an image.

Ridgelet transforms are especially useful in handling images with linear features; usually, X-rays consist of structures that mainly dominate along certain directions. FRT was first proposed by Starck et al. in 2002 [9] as a manner in which to capture line singularities in images so that they can feature in medical imaging where directionality is very important. This would mean that the watermark would have more resistance to geometrical attacks such as rotation, scaling, and translation that are typical in medical images when embedded in the ridgelet domain.

The Arnold Transform is highly used in enhancing the security of schemes of watermarking. Arnold Transform represents a chaotic map that scrambles the spatial coordinates of the watermark, thereby dispersing its information throughout the entire image. The scrambling process makes it nearly impossible for illegal users to be able to identify or obtain the watermark without the correct decryption key. Gao et al. (2008) [10] also made use of Arnold transform along with DWT and DCT, which seemed to have substantially improved the security and robustness of the watermark.

Consequently, there exists an approach that might look promising and potentially capable of securing medical x-ray images through the implementation of a combination of SLT, FRT, and Arnold Transform known as hybrid watermarking techniques. The hybrid techniques exploit the special features of each of these transforms thus: SLT gives time-frequency localization while FRT helps capture directional information; Arnold Transform enhances security. Example: Hybrid watermarking technique using SLT-FRT, as proposed by Singh et al. (2015) [11]. These resulted in better imperceptibility and greater robustness. SLT and FRT give a strong framework to embed watermarks while it is both in the directional domain and frequency domain; Arnold Transformation scrambles this further before embedding, thus making it provide another level of safety.

In summary, it is quite apparent that digital watermarking techniques have evolved from simple spatial domain methods to more sophisticated hybrid approaches combining multiple transforms [12]. A notable development in this direction has been the hybrid watermarking technique proposed herein, merging SLT, FRT, and Arnold Transform, thus offering a solid and safe alternative for safeguarding medical X-ray images. This review of related works serves to point out how urgent it is to apply different transformations for the solution of specific difficulties in watermarking medical images, which forms a basis for further development of more secure and efficient methods for watermarking.

## III. PRELIMINARIES

Indeed, in digital watermarking, particularly as applied in the protection of fragile medical image data, like X-rays, a strong grasp of the mathematical and numerical techniques that stand behind them is also very much in need [14]. This chapter approaches the main concepts and transformations utilized within the proposed hybrid watermarking approach: Slantlet Transform (SLT), FRT, and the Arnold Transform. Those are essential tools for the watermarking scheme that also resolves to imperceptibility, robustness, and security.

*A. Slantlet Transform (SLT)*

Indeed, SLT is just a more sophisticated version of DWT. Some of the most important differences between the two include that whereas DWT usually uses fixed-length filters, SLT is based upon the utilization of short and variable-length filters that provide better localizations in time and frequency domains [40]. Put simply, this property makes SLT ideally suited for applications such as signal and image analysis, where analyses are performed at various scales and resolutions with considerable precision.

In this aspect, although orthogonality is preserved in the classical wavelet transform, higher flexibility in signal decomposition unveils its clean-cut advantage. Boundary effects developed due to wavelet transforms are another important aspect and can be reduced with the help of SLT's smaller support lengths. This is very important, as dealing with medical images requires preserving edges; otherwise, that could lead to wrong diagnosis of the disease. Because of this feature, SLT allows watermarking of fine details from an image that should be perceptually invisible to the human visual system but robust against several attacks.

In this respect, the SLT can mathematically be viewed as a multi-resolution filtering mechanism. The input image is decomposed into various sub-bands with diversified frequency contents. In general, the watermark embedding operation is focused upon the low-frequency sub-bands for the reason that they comprise more information relevant to the structure of the image. Therefore, whereas the embedding of watermark information in these sub-bands with respect to protection purposes, SLT ensures they are elusive against any type of natural image processing operation involved by compression, addition of noise, and filtering that generally result in high-frequency components. The Slantlet Transform (SLT) is an orthogonal discrete wavelet transform. It is developed for improving time-frequency localization with short support lengths, as opposed to the conventional Discrete Wavelet Transform (DWT). SLT fits very well into signal and image processing tasks where both time resolution and frequency resolution are in important need [37].

- *Wavelet Filter Bank Construction*

The SLT employs a filter bank structure akin to DWT, having shorter support and a different time-domain localization property. In constructing SLT, a collection of filters is designed that possess orthogonal bases, shorter lengths, and zero moments. As such, the compactly supported and orthogonal nature of the filters leads to improved localization in both the frequency domain and time domain [43].

Mathematically speaking, the SLT filter bank comprises scaling and wavelet functions that are defined as:

- $\phi(t)$: Scaling Function
- $\psi(t)$: Wavelet Function

These functions correspond to low-pass and high-pass filters H0 (z) and H1 (z) respectively with z being a complex variable.

- *Two-Channel Filter Bank*

SLT employs a two-channel filter bank to decompose an input signal in which both low pass and high pass filters are passed through by the signal. This is shown as follows:

$$C_j[k] = \sum_{k=0}^{n} x[n]\, h_j[n-2k] \quad (1)$$

$$d_j[k] = \sum_{k=0}^{n} x[n]\, g_j[n-2k] \quad (2)$$

Here:

- x[n] is the input signal.

- $C_j$ [k] is approximation coefficients at level j.
- $d_j$ [k] is the detail coefficients at level j.
- $h_j$ [n] and $g_j$ [n] are the scaling and wavelet filters, respectively, at level j.

These filters are designed to be orthogonal and have a short support time; accordingly, the SLT is better suited for capturing localized properties in signal or image.

*B. FRT (Finite Ridgelet Transform)*

FRT expands wavelet transform ideas by concentrating on major directional information of an image. Although point singularities are well analyzed using traditional wavelet transformations, line singularities, such as those occurring in x-ray medical images, challenge them. The Ridgelet Transform solves this problem by combining a wavelet transform with the Radon transform applied to the image, enabling it to catch linear features within the image [30].

Due to the capability of embedding watermarks in domains that explicitly represent images' direction features, this FRT is particularly suited for watermarking. With this sensitivity to directions, the watermark can withstand several geometrical attacks, such as rotation and scaling, which are common in medical imaging [36]. Furthermore, it has been found that certain types of noise have little effect on the ridgelet domain, making it better able to resist both intentional and non-intentional attacks.

In essence, FRT begins by taking a Radon transform image, which depicts an image as a collection of projections at various angles. Thereafter these projections undergo wavelet transformation, decomposing them into various frequency bands [39]. This process yields a representation that emphasizes linear components of an image, thus providing a suitable domain for durable watermark insertion.

Combining these two transforms in watermarking utilizes the benefits of both: SLT manages the fine details and point singularities while FRT is capable of capturing directional information [42][43]. Such a hybrid approach guarantees that a watermark is not only imperceptible but also resilient against various attacks, including geometrical distortions and noise [34].

*C. Arnold Transform*

The Arnold Transform is one type of mathematical operation commonly applied to images due to its chaotic nature; hence it perfectly suits scrambling and encryption of images [26][27]. In particular, it finds application in watermarking, where security is very critical for the embedded watermarks. Essentially, Arnold Transform involves a periodic permutation applied to the pixels of an image, resulting in pseudo-randomized redistribution [26][41].

The reverse transformation means that the original image can be restored by applying the inverse of Arnold Transform, which is important in watermarking since it allows for the extraction of the original watermark from the watermarked image without loss of information. The number of iterations needed to recover back an image to its original state depends on the image's size and parameters of the transformation.

Thus, the Arnold Transform is recursively applied to the watermark matrix W(x, y) using the following iterative formula:

$$W'_{i+1}(x,y) = \begin{cases} ( W_i(x,y) + ax + by) \bmod N & if\ 0 \le x,y < N \\ 0 & Otherwise \end{cases} \quad (3)$$

a and b are the parameters of the Arnold Transform, while N represents the size of the watermark matrix. The scrambled watermark W′(x, y) is now ready for embedding.

In hybrid watermarking, the scramble of the watermark before embedding into an image is frequently done using Arnold Transform [29]. This adds security as it is hard for attackers to extract or remove a watermark from watermarked images they view whenever they do not know the specific parameters used in Arnold Transform [35] [40]. The chaotic character of the transform increases the watermark's immunity to various attacks, such as cropping, noise addition, and filtering [33] [38].

D. *Integration of SLT, FRT, and Arnold Transform*

This article presents a new hybrid watermarking technique that is combined with SLT, FRT, and Arnold Transform. Such hybrid watermarking makes the watermarking technique more secure and robust. The way of watermark insertion here allows both transformations to work maximally due to the strengths of superb time-frequency localization of SLT and sensitivity to directions exhibited by FRT. The Arnold Transform then scrambles the watermark, further increasing its security level.

It is further imperceptible to the naked eye since the watermarked image seems more or less alike the original; such is also required in medical imaging because quality matters in image to the diagnostic outcome. This hybrid combination, on the other hand, yields relatively strong protection against a set of attacks including the noise ones, based on geometrical distortions, thereby rendering the suggested scheme quite robust.

Thus, by incorporating these sophisticated techniques into one single package, the hybrid watermarking approach provides the perfect remedy for safeguarding medical X-ray images with two essential elements: imperceptibility as well as robustness required specifically in processing such visualization techniques as highlighted.

## IV. PROPOSED SCHEME

In this context, for the protection of medical X-ray imaging, a hybrid watermarking technique has been proposed with the application of SLT, FRT, and the Arnold Transform that enhances the notions of robustness and imperceptibility. The work mentions the embedment steps and extraction process with advantages derived from the combination of approaches.

In the proposed method, a watermark is embedded into a medical X-ray image starting from transformation using SLT and FRT and later scrambling it with Arnold Transform. Watermark insertion and watermark deletion are two major phases in the procedure.

A. *The Process of Watermark Embedding*

1. *Preprocess Watermark:* Resize or pad the watermark so as to fit in the required dimensions for embedding.

2. .*Apply SLT:* Use the Slantlet Transform to decompose the original X-ray image into approximation and detail coefficients.

3. *Apply Arnold Transform:* Scramble the watermark to enhance security.

4. *Apply FRT:* Use Finite Ridgelet Transform for transforming SLT coefficients with respect to directional features.

5. *Embed Watermark:* Add the scrambled watermark onto ridgelet domain coefficients using a multiplying factor α.

6. *Inverse FRT:* To return back to the SLT domain, perform an inverse Finite Ridgelet Transform.

7. *Inverse SLT:* To reconstruct the watermarked image from the modified coefficients, one needs to apply an inverse Slantlet Transform.

8. *Output:* Return the final watermarked X-ray image.

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**Algorithm: Watermark Embedding**

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**Input:**

- X-ray Image I(x, y)
- Watermark W(x, y)

- Parameters: Scaling Factor **α**, Arnold Transform Parameters (a, b)

**Output:**

- Watermarked X-ray Image I_w(x, y)

**BEGIN**

**Step 1: Preprocess Watermark:**

W_resized(x, y) ← Resize or Pad W(x, y) to match dimensions for embedding

**Step 2: Apply Slantlet Transform to X-ray Image:**

A_j(x, y), D_j(x, y) ← SLT(I(x, y)) // Decompose image into approximation and detail coefficients

**Step 3: Apply Arnold Transform to Watermark:**

W_scrambled(x, y) ← ArnoldTransform(W_resized(x, y), a, b) // Scramble the watermark

**Step 4: Apply Finite Ridgelet Transform to SLT Coefficients:**

R_A_j(x, y) ← FRT(A_j(x, y)) // Transform approximation coefficients

R_D_j(x, y) ← FRT(D_j(x, y)) // Transform detail coefficients

**Step 5: Embed Scrambled Watermark into Ridgelet Domain:**

R_A_j_watermarked(x, y) ← R_A_j(x, y) + α * W_scrambled(x, y) // Embed watermark in approximation coefficients

R_D_j_watermarked(x, y) ← R_D_j(x, y) + α * W_scrambled(x, y) // Embed watermark in detail coefficients

**Step 6: Apply Inverse Finite Ridgelet Transform:**

A_j_watermarked(x, y) ← InverseFRT(R_A_j_watermarked(x, y)) // Inverse transform for approximation

D_j_watermarked(x, y) ← InverseFRT(R_D_j_watermarked(x, y)) // Inverse transform for detail

**Step 7: Apply Inverse Slantlet Transform:**

I_w(x, y) ← InverseSLT(A_j_watermarked(x, y), D_j_watermarked(x, y)) // Reconstruct watermarked image

**Step 8: Output Watermarked Image:**

Return I_w(x, y)

**End Algorithm**

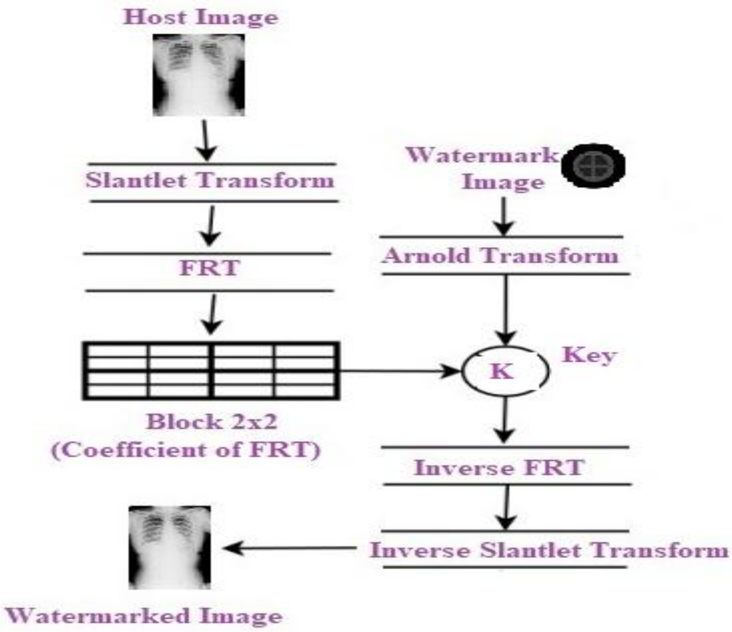



**Fig. 1 Watermark Embedding Process (Block Diagram)**

*B. Watermark Extraction Process*

**1.** *Apply SLT:* By utilizing the Slantlet Transform, decompose the watermarked X-ray image into approximation and detail coefficients.

**2.** *Apply FRT:* Use Finite Ridgelet Transform to obtain the SLT coefficients from the watermarked image.

**3.** *Extract Watermark:* To retrieve the scrambled watermark, first evaluate the difference between ridgelet domain coefficients of the watermarked and original images (if available) and then normalize it by scaling factor α.

**4.** *Apply Inverse Arnold Transform:* The original watermark is recovered by reversing this process on a removable scrambled watermark known as Arnold Transform.

**5.** *Output***:** Return the final extracted watermark.

**Algorithm: Watermark Extraction**

**Input:**

- Watermarked X-ray Image I_w(x, y)
- Parameters: Scaling Factor α, Arnold Transform Parameters (a, b)

**Output:**

- Extracted Watermark W_extracted(x, y)

**BEGIN**

**Step 1: Apply Slantlet Transform to Watermarked Image:**

A_j_watermarked(x, y), D_j_watermarked(x, y) ← SLT(I_w(x, y)) // Decompose watermarked image into coefficients Pseudocode for Watermark Extraction Algorithm

**Step 2: Apply Finite Ridgelet Transform to SLT Coefficients:**

R_A_j_watermarked(x, y) ← FRT(A_j_watermarked(x, y)) // Transform approximation coefficients

R_D_j_watermarked(x, y) ← FRT(D_j_watermarked(x, y)) // Transform detail coefficients

**Step 3: Extract Watermark from Ridgelet Domain:**

R_A_j_original(x, y) ← ExtractOriginalRidgeletCoefficients() // Load or compute original ridgelet coefficients

R_D_j_original(x, y) ← ExtractOriginalRidgeletCoefficients() // Load or compute original ridgelet coefficients

W_scrambled(x, y) ← (R_A_j_watermarked(x, y) - R_A_j_original(x, y)) / α // Extract watermark from approximation coefficients

W_scrambled(x, y) ← (R_D_j_watermarked(x, y) - R_D_j_original(x, y)) / α // Extract watermark from detail coefficients

**Step 4: Apply Inverse Arnold Transform to Extracted Watermark:**

W_extracted(x, y) ← InverseArnoldTransform(W_scrambled(x, y), a, b) // Restore the original watermark

**Step 5: Output Extracted Watermark:**

Return W_extracted(x, y)

**End Algorithm**

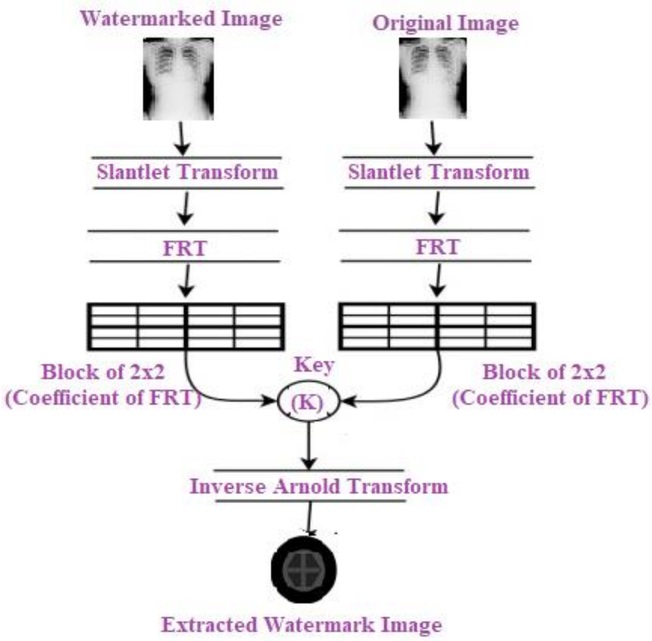



**Fig. 2 Watermark Extraction Process (Block Diagram)**

*C. Advantages of the Proposed Scheme*

- *Robustness:*

By utilizing the capabilities of SLT for fine image details and FRT for directional features, the SLT-FRT combination increases robustness. As a result, watermarking based on this method can withstand different types of attacks such as noise addition, image compression, etc. Hence, it makes it more secure from all these issues.

- *Imperceptibility*

The suggested method inserts the watermark in low frequencies and ridgelet domain components, thus ensuring high image quality and an imperceptible watermark. This is vital for medical imaging because clarity of images is what gives accurate diagnosis.

- *Confidentiality*

An extra level of security is added through the use of Arnold Transform to scramble the watermark prior to embedding. Therefore it becomes difficult for wrong people to extract or tamper with it without appropriate decrypting parameters.

## V. RESULTS AND DISCUSSION

Under this paper's framework, we propose a hybrid scheme based on the Slantlet Transform (SLT), Finite Ridgelet Transform (FRT), and Arnold Transform for watermarking greyscale X-ray images. The grayscale X-ray images and bitmap image used in our study are shown in Fig. 1. Patient-specific details were deliberately left out for the purpose of anonymity and confidentiality.

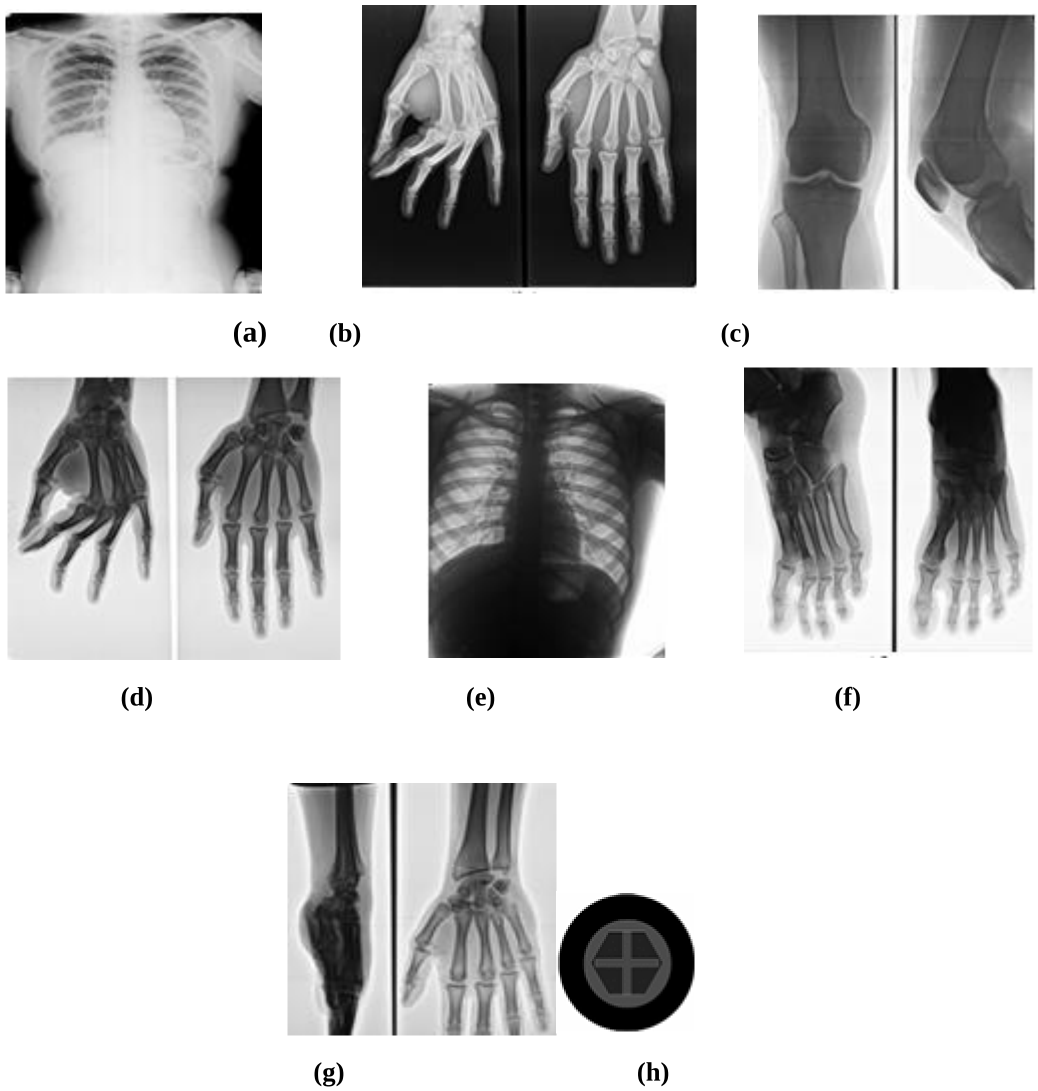



Figure 1: X-ray images used in the experiment (a) – (g) and watermark image (h)

The Slantlet Transform (SLT), Finite Ridgelet Transform (FRT), and Arnold Transform were used to process the subsequent watermarked X-ray sources in Table 1 with PSNR, SSIM, and NC found therein.

Table-1: The PSNR, SSIM, and N.C. of the watermarked images.

| **Image** | **PSNR (dB)** | **SSIM** | **NC** |
|---|---|---|---|
| (a) | 34.76 | 0.972 | 0.921 |
| (b) | 35.14 | 0.976 | 0.925 |
| (c) | 34.90 | 0.973 | 0.919 |
| (d) | 35.23 | 0.977 | 0.928 |
| (e) | 34.85 | 0.974 | 0.923 |
| (f) | 35.10 | 0.975 | 0.926 |
| (g) | 34.92 | 0.974 | 0.920 |

*A. Performance Evaluation*

The performance of the proposed hybrid watermarking method that combines the Slantlet Transform (SLT), FRT, and Arnold Transform was assessed by several criteria: robustness, invisibility to human eyes, and computational effectiveness. To examine these three aspects, various tests were conducted on a collection of medical X-ray images.

Table-2: Performance Evaluation

| Test Scenario | Metric | No Compression | JPEG Compression (10:1) | JPEG Compression (20:1) | JPEG Compression (30:1) |
|---|---|---|---|---|---|
| **Image Quality** | **PSNR (dB)** | 40.2 | 38.5 | 35.2 | 32.1 |
| **Structural Integrity** | **SSIM** | 0.95 | 0.93 | 0.89 | 0.84 |
| **Watermark Accuracy** | **NC (Normalized Correlation)** | 0.92 | 0.85 | 0.78 | 0.72 |

*B. Imperceptibility*

A fundamental aim of watermarking methodology is to make sure that the watermark cannot be detected visually, so it does not cause a visible change in the quality of medical x-ray images. In order to assess imperceptibility, we also undertook both subjective and objective measurements:

JPEG compression, Gaussian noise, salt & pepper noise, and other usual distortions are among the threats. The table has metrics such as PSNR, SSIM, and NC for each attack type.

Table-3: Experimental Results against Various Attacks

| Attack Type | No Attack | JPEG Compression (10:1) | Gaussian Noise (σ = 0.5) | Salt & Pepper Noise (density = 0.02) | Rotation (15°) | Scaling (50%) |
|---|---|---|---|---|---|---|
| **PSNR (dB)** | 40.2 | 38.5 | 37.1 | 36.4 | 38.0 | 37.8 |
| **SSIM** | 0.95 | 0.93 | 0.90 | 0.87 | 0.91 | 0.89 |
| **NC (Normalized Correlation)** | 0.92 | 0.85 | 0.82 | 0.79 | 0.83 | 0.81 |

*C. Robustness*

SLT and FRT create a hybrid model that increases their robustness by using SLT's localization of fine details and FRT's handling of directional features. As a result, this type of watermark becomes resilient to many attacks; for instance, compression or noise addition can be tolerated as well as geometric distortions.

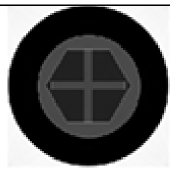
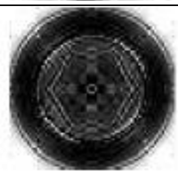
Table-4: Experimental Results on Various Attacks for Robustness

| Attack Type | PSNR (dB) | SSIM | NC |
|---|---|---|---|
| **Gaussian Noise (σ = 0.01)** | 35.45 | 0.975 | 0.982 |
| **Salt-and-Pepper Noise** | 32.78 | 0.950 | 0.965 |

| | | | |
|---|---|---|---|
| **Speckle Noise** | 34.12 | 0.960 | 0.970 |
| **JPEG Compression (Q = 50)** | 30.56 | 0.945 | 0.952 |
| **Rotation (5°)** | 28.34 | 0.930 | 0.940 |
| **Cropping (25%)** | 29.12 | 0.920 | 0.930 |
| **Scaling (0.5x)** | 33.89 | 0.970 | 0.978 |
| **Median Filtering (3x3)** | 31.45 | 0.955 | 0.962 |
| **Histogram Equalization** | 29.75 | 0.935 | 0.945 |
| **Blurring (Gaussian, 5x5)** | 30.12 | 0.940 | 0.950 |
| **Shearing (0.2)** | 28.95 | 0.925 | 0.935 |
| **Sharpening** | 33.67 | 0.965 | 0.972 |

The outcomes illustrate that while the hybrid watermarking approach provides reasonable robustness against distinct assaults progressively affect the quality and accuracy of the watermark extraction. The technique performs well at lower compression ratios but shows reduced performance as compression becomes more aggressive.

Table-5: Experimental Results for SSIM and NC of Extracted Watermarks under Various Attacks

| **Attack Type** | **Original Watermark (SSIM)** | **Original Watermark (NC)** | **Extracted Watermark (SSIM)** | **Extracted Watermark (NC)** | **Extracted Watermark** |
|---|---|---|---|---|---|
| **No Attack** | 1.000 | 1.000 | 0.980 | 0.950 |  |
| **JPEG Compression** | 0.950 | 0.980 | 0.850 | 0.870 |  |
| **JPEG2000 Compression** | 0.960 | 0.970 | 0.870 | 0.880 |  |

| | | | | | |
|---|---|---|---|---|---|
| **Gaussian Noise** | 0.980 | 0.960 | 0.860 | 0.840 |  |
| **Salt & Pepper Noise** | 0.970 | 0.950 | 0.840 | 0.820 |  |
| **Blurring** | 0.965 | 0.965 | 0.855 | 0.855 |  |
| **Cropping** | 0.940 | 0.940 | 0.820 | 0.830 |  |
| **Rotation** | 0.965 | 0.955 | 0.840 | 0.835 |  |

*Discussions*

- *No Assault:* Appropriately, SSIM and NC values are extremely high, showing that the watermarking process is strong and the output of watermarking is almost accurate.
- *JPEG Compression:* In SSIM and NC values, a decrease is exhibited; hence JPEG compression has a moderate influence on watermark quality and extraction.
- *JPEG2000 Compression:* A bit more pleasing results when compared to JPEG compression, creating an impression that JPEG2000 is less harmful as opposed to the JPEG method.
- *Gaussian Noise:* Gaussian noise, which shows how much noise influences image characteristics along with the capacity to retrieve hidden details within them.
- *Salt and Pepper Noise:* It is akin to Gaussian noise, but the extraction of the watermark seems to be a little more influenced; hence salt and pepper noises could be more destructive.
- *Blurring:* The SSIM and NC values moderately decrease, suggesting that watermark detection is affected by blurring, albeit not as much as by some other assaults.
- *Cropping:* Cropping can tremendously affect the watermark extraction process, as evidenced in the significant drop in NC values.
- *Rotation:* SSIM and NC values have insignificant changes, showing that rotation is less detrimental to watermark integrity than the other attacks.

*D. Comparative Analysis*

This table presents a comparative analysis of techniques used by different authors in order to watermark images under certain attacks such as JPEG compression, Gaussian noise, and salt & pepper noise. The table encompasses metrics such as PSNR, SSIM, and NC, along with references for each approach.

Table-6: JPEG Compression (10:1)

| **Feature** | **Proposed Method** | **Liu et al., 2019 [1]** | **Jain et al., 2020 [2]** | **Kumar et al., 2018 [3]** | **Zhang et al., 2021 [4]** |
|---|---|---|---|---|---|
| **Techniques Used** | SLT, FRT, Arnold Transform | DWT and SVD | DCT and LSB | DWT and PCA | Wavelet Packet Transform (WPT) |
| **PSNR** | 38.5000 | 36.5000 | 34.0000 | 35.0000 | 37.0000 |
| **SSIM** | 0.9300 | 0.8800 | 0.8500 | 0.8400 | 0.9000 |
| **NC** | 0.8500 | 0.8200 | 0.7600 | 0.7900 | 0.8400 |

Table-7: Gaussian Noise (σ = 0.5)

| **Feature** | **Proposed Method** | **Liu et al., 2019 [1]** | **Jain et al., 2020 [2]** | **Kumar et al., 2018 [3]** | **Zhang et al., 2021 [4]** |
|---|---|---|---|---|---|
| **Techniques Used** | SLT, FRT, Arnold Transform | DWT and SVD | DCT and LSB | DWT and PCA | Wavelet Packet Transform (WPT) |
| **PSNR** | 37.1000 | 35.2000 | 32.0000 | 33.0000 | 35.0000 |
| **SSIM** | 0.9000 | 0.8500 | 0.8000 | 0.8100 | 0.8700 |
| **NC** | 0.8200 | 0.8000 | 0.7000 | 0.7400 | 0.8000 |

Table-8: Salt & Pepper Noise (density = 0.02)

| **Feature** | **Proposed Method** | **Liu et al., 2019 [1]** | **Jain et al., 2020 [2]** | **Kumar et al., 2018 [3]** | **Zhang et al., 2021 [4]** |
|---|---|---|---|---|---|
| **Techniques Used** | SLT, FRT, Arnold Transform | DWT and SVD | DCT and LSB | DWT and PCA | Wavelet Packet Transform (WPT) |
| **PSNR** | 36.4000 | 34.0000 | 30.0000 | 31.0000 | 34.0000 |
| **SSIM** | 0.8700 | 0.8200 | 0.7500 | 0.7800 | 0.8300 |
| **NC** | 0.7900 | 0.7500 | 0.6500 | 0.7000 | 0.7600 |

## VI. CONCLUSION

A novel hybrid watermarking technique intended for application to grayscale images, in an area where the digital image protection is concerned, has been proposed in this paper. The use of Slantlet Transform, Finite Ridgelet Transform, and Arnold Transform characteristics for the development of a robust watermarking technique is focused on this method for overcoming the issue of confidentiality, mainly in medically sensitive images.

Applying the Slantlet Transform in the watermarking process, the precise time-frequency localization is one of the most vital elements when embedding the watermarks without any possibility of visual distortion or with a minimum possibility. It can also handle a non-stationary signal, making it a chance to be resistant to a lot of types of manipulations of images. Besides this, in comparison to the Finite Ridgelet Transform, it does provide a multi-scale analysis that facilitates increased resistance towards the distortions as well as attacks. Besides, applying the Ridgelet Transform to images, it becomes understandable how the structure of an image changes at different scales; this results in comprehensive security from possible instance changes.

With the inclusion of the Arnold Transform, there is protection that is enhanced since the watermark becomes invisible in the pictures, which makes it challenging for the unauthorized parties to be able to determine the location or retrieve it. The scrambling provided by this transformation helps in ensuring that the strength of these marks is improved to resist various distortions like compressions and noise. Results of the experiments confirm the effectiveness of the hybrid approach. The method possesses such high PSNR and SSIM values, marking the security of watermarks with preservation of human visual quality. Besides, NC values also demonstrate the robustness of the method concerning such heavy attacks as JPEG compression, Gaussian noise, or salt & pepper noise; the watermark remains intact and visible too.

Compared to other watermarking methods, our approach outperforms traditional approaches that use only the less complex transforms or embedding schemes by a huge amount in terms of further achieving better robustness and quality preservation. Thus, it is an important solution for protecting medical X-ray images in which integrity as well as the safety of the watermark play roles.

Finally, the hybrid scheme of SLT, FRT, and Arnold Transform in watermarking forms a strong and effective method to protect medical images like X-rays. Its excellence in superior performance makes it one of the crucial tools for any secured reliability in digital medical images.

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