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Exploring the Plane Symmetric Cosmological Model of Interacting Fluid In $f(R,T)$ Gravity

Abstract: The present manuscript presents plane symmetric cosmological model in the framework of $f(R,T)$ gravity, with an interacting field serving as the energy source. The interacting field comprises a linear coupling between an electromagnetic field, a massless scalar field and a charged perfect fluid. Four different cases have been examined, namely perfect fluid, disordered radiation, dust fluid, and dark energy. The solution of the modified gravity field equations has been generated by assuming that the deceleration parameter q is a function of the Hubble parameter H i.e.$q = b - n/H$ (where b and n are constants, and $n > 0$. While the pressure and density have been studied using the equation of state across various models. The diagnostics parameters $r(z), s(z)$ are investigated. Additionally, detailed analysis of cosmological and dynamical parameters has been conducted.

Keywords: $f(R,T)$ Theory, Interacting field, Massless scalar field, Dust fluid, Dark energy.

1. Introduction

Initially, General Relativity (GR) was successful in constructing a cosmological model and explaining the nature of the Universe. However, with the discovery of the accelerated expansion of the Universe, GR showed limitations as it couldn't account for this late-time acceleration. The current observations of type Ia supernovae suggest that the expansion of the universe is accelerating rather than slowing down (Riess[1], Perlmutter[2], Riess[3]). In response, many scientists proposed alternative theories to study the present state of the Universe by modifying or bypassing Einstein's theory of gravity. One such theory is $f(R,T)$ gravity introduced by Harko [4], where R represents the Ricci scalar and T is the trace of the stress-energy tensor. This theory formulates gravitational field equations in the metric formalism and in the equation of motion for test particles to address the late-time acceleration of the Universe. A noticeable feature of this theory is the presence of acceleration due to the coupling between matter and spacetime geometry. This phenomenon produces significant signatures and effects, which distinguishes it from other theories of gravity.

Many researchers rigorously studied $f(R,T)$ theory as an alternative to General Relativity. Nagpal [5] have constructed the cosmological model with quark and strange quark matters in $f(R,T)$ theory of gravity and describe the late-time acceleration of the Universe. Plane symmetric cosmological model with quark and strange quark matter in $f(R,T)$ theory of gravity discussed by Agrawal [6] and they have observed that the model does not approach isotropy. Sahoo [7] discussed Mixed fluid cosmological model in $f(R,T)$ gravity. Solanke [8] analyzed Two fluid cosmological models in f (R, T) theory of gravity. Singh [9] was explored Plane Symmetric Cosmological Model with Strange Quark Matter in $f(R,T)$ Gravity. Shekh [10] have studied Interacting two fluid models in modified theories of gravitation. Aktaş [11] have obtained Magnetized strange quark matter solutions in $f(R,T)$ gravity with cosmological constant. Rani [12] have examined Bianchi type-III magnetized string cosmological models for perfect fluid distribution in $f(R,T)$ gravity. Tarai [13] have studied Dynamical aspects of the magnetized anisotropic cosmological model in extended gravity. Sahoo [14] have obtained Magnetized strange quark model with Big Rip singularity in $f(R,T)$ gravity. Katore [15] have studied Two fluid cosmological models in $f(R)$theory of gravitation. Adhav [16] have discussed the Kantowski-Sachs Cosmological model with quark and strange quark matter in $f(R)$ gravity. Recognizing the significance of investigating plane symmetry and Quark matter, we endeavored to explore a plane symmetric cosmological model within both Quark and Strange quark matter frameworks. Bhardwaj [17] have discussed Evaluation of cosmological models in $f(R,T)$ gravity in different dark energy scenario. Maurya [18] considered Bianchi-I dark energy cosmological model in

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$f(R, Lm)$ −gravity. Chaubey [19] investigated the general class of Bianchi cosmological models in $f(R,T)$ gravity with dark energy in viscous cosmology. Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests discussed by Bamba [20]. Nojiri [21] have considered Unified cosmic history in modified gravity: from $f(R)$ theory to Lorentz non-invariant models. Hinshaw et al. Hinshaw [22] observed cosmological parameter results. Khade [23] have obtained Behaviour Of Bianchi Type V Model In Modified Theory Of Gravity. Reconstruction of some cosmological models in $f(R,T)$ cosmology considered by Jamil [24]. Steigman [25], demands to modify An accelerating cosmology without dark energy. Akarsu [26] have obtained Cosmological models with linearly varying deceleration parameter. Aygun [27] have discussed Strange quark matter solutions for Marder's universe in $f(R,T)$ gravity with Λ. Transit cosmological models in FRW universe under the two-fluid scenario have discussed by Garg [28]. Bennett [29] observed First-year Wilkinson microwave anisotropy probe (WMAP). Farooq [30] have observed Hubble parameter measurement constraints on the redshift of the deceleration–acceleration transition, dynamical dark energy, and space curvature.

In this paper, the work is organized as follows: In Sect. 2, we review the derivation of the field equations with variable cosmological parameter and obtain exact solutions by using parametrization of the deceleration parameter q=b$-n/H$ in Sect. 3. Sect. 4, describes the physical properties of the model. Finally, Sec. 5 is dedicated to discussion and conclusions regarding our model.

2. Gravitational Field Equation of $f(R,T)$ Gravity

Gravitational Field Equation of $f(R,T)$ Gravity

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2, \tag{1}$$

where A and B are functions of cosmic time t.

The action of $f(R,T)$ Gravity is given by

$$S = \frac{1}{2}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{2}$$

where the symbols have their usual meanings.

The gravitational field equations for $f(R,T)$ gravity is given by,

$$f\_R\{(R,T)\}\}R\_\{ij\} - ⍁\{1\}\{2\}\{f(R,T)\}g\_\{ij\} + (g\_ij\{\nabla\textasciicircum i\ \nabla\_i\} - \nabla\_i\nabla\_j)f\_R\{(R,T)\}$$
$$= (k - f\_T\{(R,T)\})\{\bar{}\{T\}\_\{ij\}\}\} - f\_T\{(R,T)\}\ \theta\_\{ij\} \tag{3}$$

where, $\theta_{ij} = g^{\alpha\beta}\frac{\partial T_{\alpha\beta}}{\partial g^{ij}}, f_R = \frac{\partial f(R,T)}{\partial R}, f_T = \frac{\partial f(R,T)}{\partial T}$

∇_i is the covarient derivative. We choose $\frac{k}{8\pi(G\}c^4}$.

Where G is the Newtonian Gravitational constant and c is the speed of light in vacuum. $\overline{T_{ij}}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m. We choose matter Lagrangian only for perfect fluid distribution as $L_m = -p$.

We assumed the model

$$f(R,T) = R + 2f(T) \tag{4}$$

Where $f(T)$ is an arbitrary function of the trace of the energy-momentum tensor and we choose $f(T) = \lambda T$, Where λ be a constant.

Now the relativistic field equations of $f(R,T)$ gravity theory for linearly coupled charged perfect fluid, source-free electromagnetic field and mass-less scalar fields are,

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = (k + 2\lambda)\overline{T_{ij}} + \lambda(\bar{T} + 2p)g_{ij} \tag{5}$$

Also, Einstein field equation for general theory of relativity is given by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} - \Lambda g_{ij} = -8\pi T_{ij} \quad (6)$$

Here, Λ is cosmological constant and we used as a dark energy source.

Comparing Eq.(5) with the Einstein field Eq.(6), we have

$$\Lambda = \lambda(\bar{T} + 2p) \quad (7)$$

We considered the source of energy of the gravitational field is an interacting

field with dark energy and observed the behaviour of the cosmological model in the presence of linearly coupled perfect fluid distribution, mass-less scalar field, and source of a free electromagnetic field. That is,

$$\overline{T_{ij}} = S_{ij} + T_{ij} + E_{ij} \quad (8)$$

where, S_{ij} is the energy-momentum tensor for perfect fluid distribution and it is given by,

$$S_{ij} = (p + \rho)u_i u_j - g_{ij}p \quad (9)$$

With

$$g^{ij}u_i u_j = 1 \quad (10)$$

Where, p, ρ, u^i are internal pressure, rest mass density and four-velocity vectors of the distribution respectively.

T_{ij} is the energy-momentum tensor for mass-less scalar field and it is given by,

$$T_{ij} = U_{,i}U_{,j} - \frac{1}{2}g_{ij}U_s U'^s \quad (11)$$

Mass-less scalar field U also satisfy

$$g^{ij}U_{;ij} = \rho_c \quad (12)$$

Where, ρ_c is the charge density, semicolon (;) and comma (,) denotes covariant

derivative and partial derivative respectively.

E_{ij} is the electromagnetic energy-momentum tensor given by

$$E_{ij} = \frac{1}{4\pi}\left[F_{i\alpha}F_j^{\alpha} - \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right] \quad (13)$$

Here F_{ij} is the electromagnetic field tensor obtained from the four potential ϕ_i,

$$F_{ij} = \phi_{i,j} - \phi_{j,i} \quad (14)$$

$$F^{ij}_{;j} = -4\pi\rho_c u^i \quad (15)$$

In the co-moving transformation system the magnetic field is considered along z −axis only, therefore non-vanishing components of electromagnetic fields F_{ij} are only F_{12} and F_{21}. Also, we have electromagnetic field tensor is anti-symmetric.

The first set of Maxwell equation are,

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \quad (16)$$

leads to

$$F_{12} = constant = M \quad (17)$$

Now from Eq.(8), (9), (13) for the metric (1), we have

$$\bar{T} = -\dot{U}^2 - 3p + \rho \quad (18)$$

Now, by using $F_{12} = const. = M$ and $u^4 \neq 0$, from Eqn.(15) we have, charge density is zero $(\rho_c = 0)$

From Eq.(5) and Eq.(11), we get

$$\frac{\ddot{U}}{\dot{U}}+2\frac{\dot{A}}{\dot{A}}+\frac{\dot{B}}{\dot{B}}=0 \tag{19}$$

From Eq.(1) and Eq.(6), we have

$$U=\frac{\beta}{4}(1+z)^4+\gamma \tag{20}$$

The field equations of $f(R,T)$ Eq.(5), for the metric Eq.(1) can be reduced as,

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-(k+2\lambda)\left[\frac{M^2}{8\pi A^4}-\frac{\dot{U}^2}{2}-p\right]-\Lambda \tag{21}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\ddot{A}}{A}=-(k+2\lambda)\left[-\frac{M^2}{8\pi A^4}-\frac{\dot{U}^2}{2}-p\right]-\Lambda \tag{22}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{B}}{AB}=-(k+2\lambda)\left[-\frac{M^2}{8\pi A^4}+\frac{\dot{U}^2}{2}+\rho\right]+\Lambda \tag{23}$$

Here dot for differentiation with respect to time t.

Average scale factor is,

$$a(t)=(A^2B)^{1/3} \tag{24}$$

Spatial Volume is,

$$V=A^2B \tag{25}$$

Directional Hubble parameters are,

$$H_x=H_y=\frac{\dot{A}}{A},\ H_z=\frac{\dot{B}}{B} \tag{26}$$

Average Hubble Parameter is,

$$H=\frac{1}{3}\left[2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right] \tag{27}$$

According to the proposed law, the variation of the mean Hubble parameter for plane symmetric metric is given by

$$\mathrm{H}=\mathrm{k}(\mathrm{A}^2\mathrm{B})^{-\mathrm{m}/3}, \tag{28}$$

$k>0,\ m\geq 0$ are constant.

3. Solution of the field equations

To be more precise, our approach in this paper is to derive the Hubble parameter H, from the following deceleration parameter, $q=b-\frac{n}{H}$. We recall that for an accelerating expansion $q<0$, for a decelerating expansion $q>0$ while for constant rate expansion $q=0$. In Tiwari et al.[31-34], the authors proposed a linear relation between the deceleration and Hubble parameters, we assume a relation in the form

$$q=b-\frac{n}{H} \tag{29}$$

$n>0, b$ are constants.

$$H=\frac{n}{(cne^{-nt}+b+1)} \tag{30}$$

where c is an integration constant and again integrating the above equation, we have

$$a=k_1(b+1)(e^{nt}-1)^{1/(b+1)} \tag{31}$$

where k_1 is a integrating constant. Here, a point type singularity occurs at $t = 0$ with the choice of constant c as $c = -\frac{(b+1)}{n}$. With this choice, the Hubble parameter H and the scale factor a is given as

$$H = \frac{ne^{nt}}{(b+1)(e^{nt}-1)} \tag{32}$$

$$a = k_1(b+1)(e^{nt}-1)^{\frac{1}{(b+1)}} \tag{33}$$

where k_1 is a constant. We should mention that $b = -1$ is a singular point. Therefore, the priors of the model's parameters must be carefully chosen. We have allowed the parameters to vary as $b \in (0, 1)$, $k_1 \in (0,1)$. The deceleration parameter is written in terms of cosmic time t

$$q = -1 + \frac{(b+1)}{e^{nt}} \tag{34}$$

The sign of deceleration parameter in our model changes at $t = \frac{1}{n}\log(b+1)$. In addition, using the relation between the redshift and the scale factor of the Universe $a(t) = (1+z)^{-1}$, we can define the relation between the cosmic time and redshift as

$$t = log\left[\left(\delta(1+z)\right)^{-(b+1)} + 1\right]^{\frac{1}{n}} \tag{35}$$

The Hubble and deceleration parameters are written in terms of redshift as

$$H(z) = \frac{n}{(b+1)}\left[1 + \left(\delta(1+z)\right)^{(b+1)}\right] \tag{36}$$

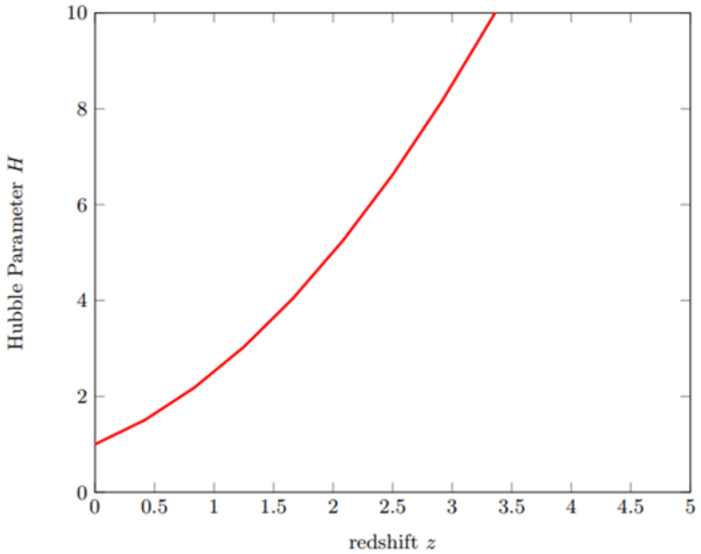

Figure 1. Hubble parameter vs. redshift

The Hubble parameter for $z = 0$ is,

$$H(z=0) = H_0 = \frac{n}{(b+1)}\left[1 + \delta^{(b+1)}\right] \tag{37}$$

The Hubble rate function takes the form

$$H(z) = \frac{H_0}{\left[1+\delta^{(b+1)}\right]}\left[1 + \left(\delta(1+z)\right)^{(b+1)}\right] \tag{38}$$

The expression for Hubble parameter in term of redshift (z) as depicted in fig. 1 The numerous data are available in terms of redshift z to easily be compared with theoretical studies in terms of redshift. We find the best fit values for the model parameters $b = 0.444, n = 40.73$ and $\delta = 1.159$. Now, Eq. (36) becomes

$$H(z) = 28.2063 + 34.9047(1+z)^{1.444}.$$

The current value of Hubble's parameter H for the obtained model, have calculated as $H_0 = 63.111 \pm 11.54$Km/s/Mpc. Fig 1. shows the best fit curve of the Hubble's parameter versus red-shift z using Pacif [35], Sharov[36]. Hubble's parameter measurements.

The sign of jerk parameter is important in that it shows the change in the dynamics of the universe. r with a positive sign points out the existence of a period in which the expansion of the universe changes. Both of r and s are important as they show the deviation of a dark energy model from the Λ CDM model Capozziello [37], [38]. The present values of these parameters in Λ CDM model are $r = 1$ and $s = -0.35$.

The expansion scalar θ, defined by $\theta = 3H$, is found as

$$\theta = \frac{3H_0}{[1+\delta^{(b+1)}]}\left[1 + \left(\delta(1+z)\right)^{(b+1)}\right] \tag{39}$$

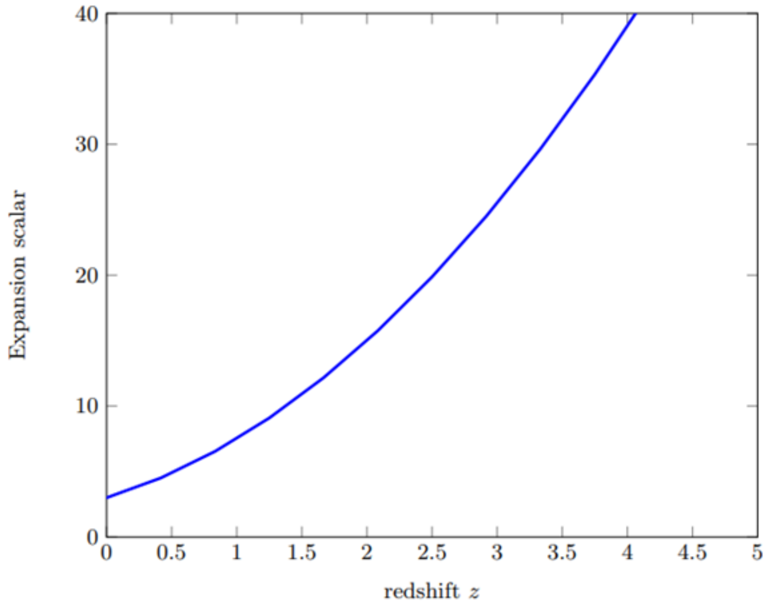

Figure 2. Expansion scalar vs. redshift

The Expansion Scalar initially small with redshift z then increases with redshift z. It is clear from the fig. 2 that the Expansion Scalar remains in the positive domain throughout the entire history of the universe.

The directional Hubble parameters as defined in (26) are found as

$$H_x = \frac{3\alpha H_0}{(2\alpha+1)[1+\delta^{(b+1)}]}\left[1 + \left(\delta(1+z)\right)^{(b+1)}\right] \tag{40}$$

$$H_z = \frac{3H_0}{(2\alpha+1)[1+\delta^{(b+1)}]}\left[1 + \left(\delta(1+z)\right)^{(b+1)}\right] \tag{41}$$

On manipulating the result by using (40) and (41),we get the following exact expressions for the scale factors:

$$A = exp\{\frac{3\alpha H_0\left[\delta(b+2)z+\left(\delta(1+z)\right)^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta[1+\delta^{(b+1)}]}\} \tag{42}$$

$$B = exp\{\frac{3H_0\left[\delta(b+2)z+\left(\delta(1+z)\right)^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta[1+\delta^{(b+1)}]}\} \tag{43}$$

The shear scalar σ^2 is found as

$$\sigma^2 = 3\left[\frac{H_0(\alpha-1)}{(2\alpha+1)[1+\delta^{(b+1)}]}\right]^2\left[1 + \left(\delta(1+z)\right)^{(b+1)}\right]^2 \tag{44}$$

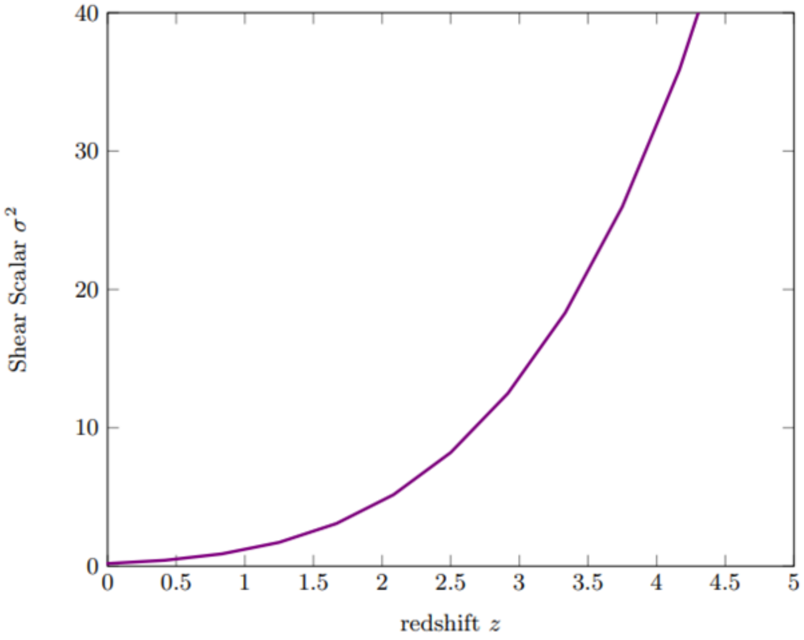

Figure 3. Shear scalar vs. redshift

The anisotropy parameter of the expansion A_m is defined as

$$A_m = 2\left[\frac{\alpha-1}{2\alpha+1}\right]^2 \tag{45}$$

The shear scalar is small at the initial epoch and becomes infinite as redshift increases as depicted in fig. 3. For our model, The statefinder parameters of our model in terms of redshift z are calculated as,

$$r = 1 - \frac{3(b+1)}{\left[1+(\delta(1+z))^{-(b+1)}\right]} + \frac{(b+1)^2}{n\left[1+(\delta(1+z))^{-(b+1)}\right]^2} \tag{46}$$

$$s = \frac{2(b+1)[-3n[1+(\delta(1+z))^{-(b+1)}]+b+1]}{[1+(\delta(1+z))^{-(b+1)}[-3[1+(\delta(1+z))^{-(b+1)}]+2(b+1)]} \tag{47}$$

However, it is important to assess the jerk parameter for our model in order to understand its deviation from the standard Λ CDM model. The jerk parameters of our model in terms of redshift z is,

$$j = 1 - \frac{3(b+1)}{[1+(\delta(1+z))^{-(b+1)}]} + \frac{2(b+1)^2}{[1+(\delta(1+z))^{-(b+1)}]^2} - \frac{(b+1)^2}{2+(\delta(1+z))^{(b+1)}+(\delta(1+z))^{-(b+1)}} \tag{48}$$

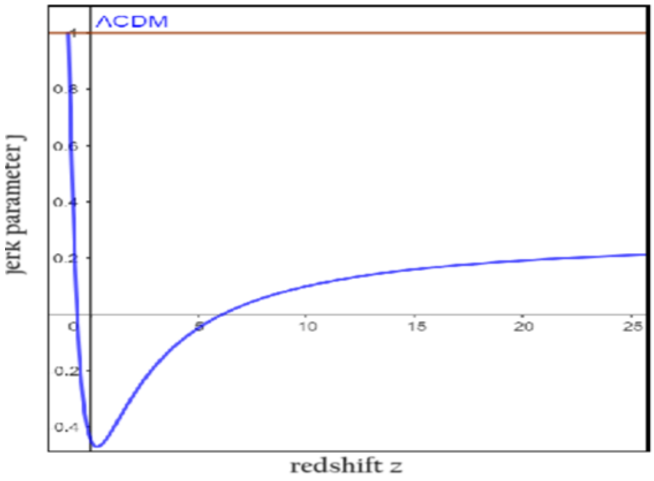

Figure 4. Jerk parameter vs. redshift

For a flat Λ CDM model, the jerk parameter j has the value $j = 1$.Our model approaches the Λ CDM model for $b = \frac{1}{5}, z \to -1$.

This study aims to analyze a cosmological model through mathematical methods to understand how the Universe behaves on a large scale. The model considers various geometrical and physical factors to determine the dynamics of the Universe. There are some cosmological parameters that can quantify the behavior of the Universe. The key parameters include the Hubble parameter $H(t)$ and the deceleration parameter $q(t)$, which are fundamental in understanding the Universe's evolution. which describe the dynamics of the Universe. We plot a graph showing the relationship between the deceleration parameter q and the redshift z. We select the model parameter n appropriately to achieve a transition redshift (z) that demonstrates a shift from early deceleration to later acceleration. The expression for the deceleration parameter in relation to the redshift z can be formulated. Choosing the model parameter n suitably, $q(z)$ can be plotted for a close view to discuss the behavior of the deceleration parameter as shown in fig.4.

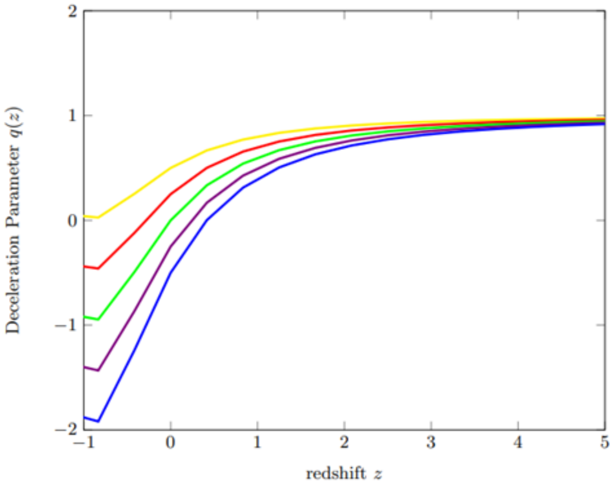

Figure 5. Deceleration parameter vs. redshift

The deceleration parameter q shows a positive value at high redshifts and a negative value at low redshifts, within the range of $0.5 < n < 1.5$ for the model parameter n. The shift from deceleration to acceleration in the evolution of the Universe is influenced by changes in the model parameter n.When n ≤ 1, the model indicates perpetual acceleration. The plot shows the phase transition redshift (z) for various values of n in the feasible range, $n \in (0.5, 1.5)$.

4. Properties of the Model

4.1 Zeldovich Fluid Model: $p = \rho$

We assume $p = \boldsymbol{\rho}$ In this case, we obtain, the pressure (p) cosmological constant (Λ) in terms of redshift (z) is as follows:

$$p = \rho = \frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+(\delta(1+z))^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta\left[1+\delta^{(b+1)}\right]}\}} - \frac{9\alpha(\alpha+2)H_0^2\left(1+(\delta(1+z))^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)} \tag{49}$$

$$\Lambda = -\lambda\beta^2(1+z)^6 \tag{50}$$

In this model, we observed that energy density ρ is positive and decreasing function of redshift z. In the initial phases of the early universe for a very small redshift, the pressure (p) and energy density ρ attains a very large value and approaches to $\frac{M^2}{8\pi exp\{\frac{12\alpha H_0[\delta(b+2)z]}{(2\alpha+1)(b+2)\delta\left[1+\delta^{(b+1)}\right]}\}} - \frac{9\alpha(\alpha+2)H_0^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2}$ in the future as z approaches to -1.

Cosmological constant remains positive and increasing in nature throughout the cosmic evolution for negative value of λ. As $z \rightarrow 0$, $\Lambda \rightarrow -\lambda\beta^2$ and for $z \rightarrow -1$, $\Lambda \rightarrow 0$.

4.2 Disordered Radiation Model: $\rho = 3p$

We assume $\rho = 3p$. In this case, we obtain, The pressure (p), energy density ρ and cosmological constant (Λ) in terms of redshift (z) is as follows:

$$p = \frac{(k+2\lambda)}{(3k+4\lambda)}\{\frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+(\delta(1+z))^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta\left[1+\delta^{(b+1)}\right]}\}} - \frac{9\alpha(\alpha+2)H_0^2\left(1+(\delta(1+z))^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{51}$$

$$\rho = \frac{3(k+2\lambda)}{(3k+4\lambda)}\{\frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+(\delta(1+z))^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta[1+\delta^{(b+1)}]}\}}$$

$$- \frac{9\alpha(\alpha+2)H_0^2\left(1+(\delta(1+z))^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{52}$$

We obtained cosmological constant Λ is given by the equation

$$\Lambda = -\lambda\beta^2(1+z)^6 + \frac{2\lambda(k+2\lambda)}{(3k+4\lambda)}\{\frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+(\delta(1+z))^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta\left[1+\delta^{(b+1)}\right]}\}} - \frac{9\alpha(\alpha+2)H_0^2\left(1+(\delta(1+z))^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} -$$

$$\frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{53}$$

In this model, we observed that as $z \rightarrow -1$, $\alpha \rightarrow 0$, the pressure $(p) \rightarrow \frac{(k+2\lambda)}{(3k+4\lambda)}\frac{M^2}{8\pi}$, energy density $\rho \rightarrow \frac{3(k+2\lambda)}{(3k+4\lambda)}\frac{M^2}{8\pi}$ and Cosmological constant $\Lambda \rightarrow \frac{2\lambda(k+2\lambda)}{(3k+4\lambda)}\frac{M^2}{8\pi}$.

4.3 Dust Fluid Model: $p = 0$

We assume $p = 0$. In this case, we obtain, The energy density ρ and cosmological constant (Λ) in terms of redshift (z) is as follows:

$$\rho = \frac{(k+2\lambda)}{(k+\lambda)}\{\frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+(\delta(1+z))^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta\left[1+\delta^{(b+1)}\right]}\}} - \frac{9\alpha(\alpha+2)H_0^2\left(1+(\delta(1+z))^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{54}$$

$$\Lambda = -\lambda\beta^2(1+z)^6 + \frac{\lambda(k+2\lambda)}{(k+\lambda)}\{\frac{M^2}{8\pi exp\frac{12\alpha H_0[\delta(b+2)z+(\delta(1+z))^{(b+2)}]}{(2\alpha+1)(b+2)\delta[1+\delta^{(b+1)}]}}$$

$$- \frac{9\alpha(\alpha+2)H_0^2(1+(\delta(1+z))^{(b+1)})^2}{(k+2\lambda)(2\alpha+1)^2[1+\delta^{(b+1)}]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{55}$$

In dust fluid model,we observed that for $z \rightarrow -1$, $\alpha \rightarrow 0$, The energy density $\rho \rightarrow \frac{(k+2\lambda)}{(k+\lambda)}\frac{M^2}{8\pi}$ and Cosmological constant $\Lambda \rightarrow \frac{\lambda(k+2\lambda)}{(k+\lambda)}\frac{M^2}{8\pi}$.

4.4 Dark Energy Model: $p = -\rho$

For dark energy EoS parameter $\omega = -1$, We assume $p = -\rho$ the energy density ρ and cosmological constant (Λ) in terms of redshift (z) of the model are as follows:

$$\rho = \frac{(k+2\lambda)}{k}\{\frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+\left(\delta(1+z)\right)^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta[1+\delta^{(b+1)}]}\}} - \frac{9\alpha(\alpha+2)H_0^2\left(1+\left(\delta(1+z)\right)^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{56}$$

$$\Lambda = -\lambda\beta^2(1+z)^6 + \frac{2\lambda(k+2\lambda)}{k}\{\frac{M^2}{8\pi exp\{\frac{12\alpha H_0\left[\delta(b+2)z+\left(\delta(1+z)\right)^{(b+2)}\right]}{(2\alpha+1)(b+2)\delta[1+\delta^{(b+1)}]}\}} - \frac{9\alpha(\alpha+2)H_0^2\left(1+\left(\delta(1+z)\right)^{(b+1)}\right)^2}{(k+2\lambda)(2\alpha+1)^2\left[1+\delta^{(b+1)}\right]^2} - \frac{(k+4\lambda)\beta^2(1+z)^6}{2(k+2\lambda)}\} \tag{57}$$

For dark energy model, we observed that, for $z \to -1, \alpha \to 0$, The energy density $\rho \to \frac{(k+2\lambda)M^2}{8\pi k}$ and Cosmological constant $\Lambda \to \frac{2\lambda(k+2\lambda)M^2}{8\pi k}$.

5. Discussion and Conclusion

In this paper, we have examined a plane symmetric cosmological model that considers an interacting fluid with a cosmological constant acting as a form of dark energy within the framework of the $f(R,T)$ theory of gravity. The term "interacting fluid" refers to the formation of a relativistic field equation of the $f(R,T)$ theory of gravity for a plane symmetry cosmological model that is linearly coupled with a charged perfect fluid, mass-less scalar field, and electromagnetic field. To get exact solutions of Einstein's field equations, we assumed that the linear relation between deceleration parameter (q) and Hubble parameter (H), which yields a scale factor a as given in Eq. (31). We have explored four distinct models within this framework, which include the Zeldovich fluid, Disordered radiation, Dust fluid and Dark energy.

In order to study a cosmological model, we considered a variable deceleration pararmeter q. From the Eq. (32)-(34) The behavior of the geometrical parameters a, H and q at two extremities have been analyzed. As $t \to 0$, $H \to 0, a \to 0$, and $q \to b$. However, as $t \to \infty$, $q \to -1$. This shows that the universe in the model has a transition from an earlier decelerating phase to the present accelerating phase and also shows the largest negative value of the deceleration parameter (q), and hence the fastest rate at which the universe is undergoing expansion.

We have found the different stages of evolution of the deceleration parameter based on the chosen H parameterization. By examining the expression for $q(z)$, we have determined that the deceleration parameter falls within the range of $q \in [n-1,-1]$, indicating a transition from deceleration to acceleration due to the model parameter n being greater than 1. We can observe the shift from decelerating to accelerating phases in the Universe depending on how the model parameter n varies, as illustrated in fig.5. An increase in the values of n from 0.5 to 1.5 could potentially delay the redshift z associated with the phase transition.

It was noted that the characteristics of pressure and density exhibit varying trends across distinct models. In the case of a stiff fluid, the pressure decreases over redshift z; however, at the initial moment, the pressure is infinite. For disordered radiation, the pressure is infinity at the initial epoch. For dust fluid we have seen pressure is zero therefore we observed density of the model. We have observed density for dust fluid is increasing. For dark energy cosmological model, pressure is clearly negative in nature. It is noteworthy that we have found varying cosmological constant with respect to redshift z.

From Eq. (46) and (47), we observe that, r and s are independent of redshift (z). Equation (48) gives a positive value for an appropriate choice of $b = \frac{1}{5} and\ z \to -1$. Thus, there is a smooth transition of the Universe from decelerating to accelerating phase of the Universe.

The jerk parameter (j) has also been found, which predicts that the universe in the model tends to the Λ CDM model at late times. The results obtained and the observed behaviour of the models agree with the recent observational facts of cosmology Nagpal [39] and Sahoo [40].

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