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# Effect of Suspended Particle on Couple Stress Ferromagnetic Micropolar Fluid Heated from Below in Porous Medium



## Abstract

This study explores how the stability of couple stress Ferromagnetic Micropolar Fluid heated from below inside a porous medium is affected by suspended particles. The research investigates how suspension particles affect fluid behavior and heat transfer properties inside a magnetic field by including them. Key results show that under certain circumstances the presence of suspended particles notably changes convection patterns and improved thermal stability. These observations deepen one's knowledge of micropolar fluid mechanics, which could have uses in geophysics, industrial operations, and sophisticated cooling systems.

**Keywords:** Suspended solids, Couple Stress Fluid, Ferromagnetic micropolar solution, Thermal convection, Porous substrate, Stability of heat transfer, Magnetic fluid dynamic properties, Micropolar mechanics, Thermal disorder, Fluid dynamic in porous systems

## 1. Introduction

### 1.1 Background and Motivation

Unlike Newtonian fluids, micropolar fluids display micro-rotational effects, making them appropriate for simulation of sophisticated fluids including blood, polymeric suspensions, and liquid crystals [1]. Ferromagnetic fluids, on the other hand, are colloidal suspensions of magnetic particles in a carrier fluid and show distinctive magneto-hydrodynamic behaviour [2]. Further complicated by the existence of suspended particles and a porous medium, this research pays attention to a particular kind of micropolar fluid: couple stress ferromagnetic fluid.

Several engineering uses, including those listed below, depend on knowledge of how these complex fluids behave:

**Geophysics:** study of magma flow inside the crust of the Earth.

**Bioengineering:** Blood flow in human body modelled.

**Chemical Engineering:** Design of effective filtration systems and heat exchangers in chemical engineering.

**Materials Science:** Developing new substances with improved thermal and magnetic properties characterizes materials science.

### 1.2 Objectives of the Study

This study aims first to:

- To check a flammic fluid layer warmed from beneath in a thin-medium for stability a couple pressures.
- To study how the start of convection in the fluid is affected by suspended particulates.
- To establish the important Rayleigh number—the point at which convection instability begins—that characterizes this process.

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- To study how the fluid's stability properties are affected by several variables, among them magnetic field strength, porosity, and particle concentration.

### 1.3 Scope of the Research

The start of convection in a horizontal layer of couple stress magnetically saturated with a porous medium will be studied using linear stability analysis in this research. The liquid is thought to be warmed from underneath and include suspended particles. The evaluation will take into account how the stability of the system is affected by different physical properties.

## 2. Literature review

### 2.1 Couple Stress Fluid

Couple Stress Fluids, proposed by Eringen [3], are a subclass of fluids with capabilities of exhibiting micro-rotational effects, which make them convenient to be used for model systems like complex fluids with internal microstructures. There were a lot of studies on the stability and flow characteristics of couple stress fluids depending on various conditions [4].

### 2.2 Ferromagnetic and Micropolar Fluids Properties

The unique ferromagnetic property of ferrofluids is their use in intense studies over the years. The combination of ferromagnetic and micropolar fluid properties involves a further degree of complexity in the problem and, hence, gives rise to exciting magnetohydrodynamic phenomena.

### 2.3 Suspended particles in fluid dynamics

The presence of suspended particles substantially affects the stability and flow behavior of fluids. Many works have focused on the effects of particle concentration, size, and shape on the onset of convection within various fluid systems.

### 2.4 Heat Transfer in Porous Media

Heat transfer in porous media is vital to many engineering applications, such as geothermal energy extraction and filtration processes. Several works were devoted to stability and flow characteristics of fluids in porous media that account for factors like permeability and porosity.

## 3. Mathematical Formulation

It is in this direction that the study aims to analyze a couple stress ferromagnetic micropolar fluid heated from below flowing in a fluid-porous medium with the capability of influencing suspended particles. This section plunges into the mathematical castigations of the subject, giving the basic equations governing dynamics, equations governing couple stress ferromagnetic micropolar fluids, the incorporation of matter suspension, and boundary conditions with laid assumptions in order to bring more clarity.

### 3.1 Basic Equations of Motion

The laws of motion are created from some basic rules of conservation, such as mass, momentum (linear and angular), the torque of the rotor, and the spirit of the mechanics, the thermal effect, magnetization, and the touch stress in actions. Now let us examine or explain gently each of these equations,

#### 3.1.1. Conservation of Mass (Continuity Equation)

In the case of an incompressible fluid:

$$\nabla \cdot \vec{v} = 0,$$

where  $\vec{v}=(u,v,w)$  represents the velocity components in x,y,z -directions, respectively.

### 3.1.2. Conservation of Momentum

The simplified Navier-Stokes equation is supplemented by additional terms to impose the stress related to the couple and magnetic forces. In a porous medium, the Darcy-Brinkman model introduces a drag term related to velocity:

$$\rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \mu \nabla^2 \vec{v} + k\mu \vec{v} + \nabla \cdot \tau_{ij} + \rho \vec{g} + \vec{F}_m,$$

where:

- $\rho$ : Fluid density,
- $\mu$ : Dynamic viscosity,
- $k$ : Permeability of the porous medium,
- $\tau_{ij}$ : Couple stress tensor,
- $\vec{F}_m$ : Magnetic force, defined as:

$$\vec{F}_m = \mu_0 (\vec{M} \cdot \nabla) \vec{H},$$

With  $\mu_0$ : Permeability of free space,  $\vec{M}$ : Magnetization, and  $\vec{H}$ : Magnetic field.

### 3.1.3. Angular Momentum Conservation

The couple stress fluid introduces a micro-rotational field  $\vec{N}$ , governed by:

$$\rho \left( \frac{\partial \vec{N}}{\partial t} + \vec{v} \cdot \nabla \vec{N} \right) = \gamma \nabla^2 \vec{N} + \nabla \cdot \tau_{ij} + \nabla \times \vec{H},$$

Where  $\gamma$  is the spin viscosity.

## 4. Energy Equation

The energy equation governs heat transfer, with thermal conductivity  $k$ , generation  $Q$ , and contributions from viscous and magnetic dissipation:

$$\rho c_p \left( \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = k \nabla^2 T + Q, \quad (3.1.4)$$

where:

- $T$ : Temperature,
- $c_p$ : Specific heat capacity,
- $Q$ : Heat generation from viscous and magnetic forces:

$$Q = \frac{\mu}{2} (\nabla \vec{v} + \nabla \vec{v}^T)^2 + \mu_0 \chi_m \frac{\partial \vec{H}}{\partial t}.$$

### Non-Dimensionalization

Introduce the following dimensionless variables:

$$x' = \frac{x}{d}, t' = \frac{\alpha t}{d^2}, T' = \frac{T - T_b}{\Delta T}, Ra = \frac{g \beta d^3 \Delta T}{\nu \alpha},$$

where  $Ra$  is the Rayleigh number,  $\Delta T$ : temperature difference,  $\beta$ : thermal expansion coefficient, and  $\nu$ : kinematic viscosity.

The non-dimensionalized equations become:

**1. Continuity:**

$$\nabla' \cdot \vec{v}' = 0. \quad (3.1.5)$$

**2. Momentum:**

$$\frac{\partial \vec{v}'}{\partial t'} + Ra T' = -\nabla' p' + \nabla'^2 \vec{v}' - \frac{\mu'}{k'} \vec{v}' + M \nabla'^2 \vec{H}', \quad (3.1.6)$$

Where  $M = \frac{\mu_0 \chi m}{\rho \nu}$ : Magnetic coupling parameter.

**3. Energy:**

$$\frac{\partial T'}{\partial t'} + \vec{v}' \cdot \nabla' T' = \nabla'^2 T'. \quad (3.1.7)$$

**4. Micropolar Angular Momentum:**

$$\frac{\partial \vec{N}'}{\partial t'} + \vec{v}' \cdot \nabla' \vec{N}' = \nabla'^2 \vec{N}' + \nabla' \times \vec{H}'. \quad (3.1.8)$$

**3.2. Couple stress ferromagnetic micropolar fluids: Equations**

This section elaborates the governing equations for convective heat and suspended particle behavior of a couple stress ferromagnetic micropolar fluid. The contributions of micro-rotational fields, thermal gradients, magnetic forces, and couple stresses are all accounted for. This analysis is applied to a permeable medium heated from below, which introduces more Darcy-Brinkman drag terms. It presents the equations, provides numerical solutions and graphical representation.

**1. Physical Model and Assumptions.**

- Incompressible fluid flows in a 2D rectangular cavity.
- Couple stresses are a result of the micro-rotational effects that occur in the fluid.
- The application of a uniform vertical magnetic field results in the creation of magnetization.
- Heat is applied from below, resulting in convection due to buoyancy.
- Momentum, momentum, and viscosity are influenced by suspended particles.

**2. Governing Equations.**

The flow, temperature, particle dynamics and micro-rotation are the equations that are derived from this.

- For couple stress fluids Navier–Stokes equation has been modified.
- The conservation of angular momentum in micro-rotation effects,
- The equation for transferring heat in thermal convection,
- Porous media is referred to as a Darcy-Brinkman drag.
- Magnetization force terms.

**2.1 Momentum Equation**

The momentum equation (Boussinesq approximation) is:

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right) = -\nabla p + \mu \nabla^2 \vec{v} + \frac{\mu}{k} \vec{v} + \nabla \cdot \tau_{ij} + \rho g \beta (T - T_0) \hat{z} + \vec{F}_m, \quad (3.2.1)$$

where:

- $\tau_{ij} = \eta_c \left( \nabla^2 \vec{N} - \frac{\vec{N}}{l_c^2} \right)$ : Couple stress tensor,
- $\vec{F}_m = \mu_0 (\vec{M} \cdot \nabla) \vec{H}$ : Magnetic force,
- $\rho g \beta (T - T_0)$ : Buoyancy force,
- $\frac{\mu}{k}$ : Darcy-Brinkman drag term in porous media.

## 2.2 Energy Equation

Thermal convection is described as:

$$\rho c_p \left( \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = k \nabla^2 T + Q, \quad (3.2.2)$$

where:

$$Q = \frac{\mu}{2} (\nabla \vec{v})^2 + \mu_0 \chi_m \frac{\partial \vec{H}}{\partial t}: \text{Heat generation from viscous and magnetic effects.}$$

## 2.3 Micropolar Angular Momentum Equation

Micro-rotations  $\vec{N}$ : are governed by:

$$\frac{\partial \vec{N}}{\partial t} + \vec{v} \cdot \nabla \vec{N} = \gamma \nabla^2 \vec{N} + \nabla \cdot \tau_{ij} + \nabla \times \vec{H}, \quad (3.2.3)$$

Where  $\gamma$  is the spin viscosity.

## 2.4 Suspended Particle Dynamics

Particles alter momentum and energy equations:

### 1. Modified Viscosity:

$$\mu_{eff} = \mu \left( 1 + \frac{5}{2} \phi \right),$$

Where  $\phi$  is the particle volume fraction.

### 2. Thermal Conductivity:

$$k_{eff} = k(1 + \beta \phi).$$

## 2.5 Dimensionless Governing Equations

Using dimensionless variables:

$$x' = \frac{x}{d}, t' = \frac{\alpha t}{d^2}, T' = \frac{T - T_0}{\Delta T},$$

and the Rayleigh number  $Ra = \frac{g \beta d^3 \Delta T}{\nu \alpha}$  the equations become:

### 1. Dimensionless Momentum:

$$\frac{\partial \vec{v}'}{\partial t'} + Ra T' = -\nabla' p' + \nabla'^2 \vec{v}' - \frac{\mu'}{k'} \vec{v}' + M \nabla'^2 \vec{H}'. \quad (3.2.4)$$

**2. Dimensionless Energy:**

$$\frac{\partial T'}{\partial t'} + \vec{v}' \cdot \nabla' T' = \nabla'^2 T'. \quad (3.2.5)$$

**3. Dimensionless Micropolar Angular Momentum:**

$$\frac{\partial \vec{N}'}{\partial t'} + \vec{v}' \cdot \nabla' \vec{N}' = \nabla'^2 \vec{N}' + \nabla' \times \vec{H}'. \quad (3.2.6)$$

**3.3 Incorporation of Suspended Particles**

A couple stress ferromagnetic micropolar fluid in a porous medium heated from below demonstrates altered behavior due to suspended particles. Suspended particles change fluid properties such as viscosity and density while modifying thermal conductivity and heat capacity which affects momentum transport alongside heat transfer and micro-rotational behavior.

The section develops governing equations with suspended particles included, solves these equations mathematically and displays numerical results along with graphical data.

**1. Effect of Suspended Particles on Fluid Dynamics**

Suspended particles **enhance or suppress convection**, depending on their properties. The following effects are considered:

**1. Density Modification:**

The effective density of the fluid-particle mixture is:

$$\rho_{eff} = (1 - \phi)\rho_f + \phi\rho_p,$$

Where:

- $\phi$  is the particle volume fraction,
- $\rho_f$  is the fluid density,
- $\rho_p$  is the particle density.

**2. Viscosity Enhancement:**

The effective viscosity follows Einstein's relation

$$\mu_{eff} = \mu\left(1 + \frac{5}{2}\phi\right).$$

**3. Thermal Conductivity Enhancement:**

The effective thermal conductivity of the fluid-particle suspension is given by:

$$k_{eff} = k(1 + \beta\phi),$$

Where  $\beta$  is the particle conductivity enhancement factor.

**4. Momentum Equation Modification:**

Additional drag forces due to suspended particles modify the momentum equation.

**2. Governing Equations with Suspended Particles****2.1 Modified Momentum Equation**

The momentum equation, incorporating suspended particles, is:

$$\rho_{eff} \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \mu_{eff} \nabla^2 \vec{v} + \rho_{eff} g \beta (T - T_0) \hat{z} + \vec{F}_m - \gamma (\vec{v} - \vec{v}_p),$$

Where:

- $\vec{F}_m$  is the magnetic force,
- $\gamma$  is the drag coefficient between fluid and particles,
- $\vec{v}_p$  is the particle velocity.

## 2.2 Particle Transport Equation

The concentration of suspended particles evolves according to the **advection-diffusion equation**:

$$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = D_p \nabla^2 \phi,$$

Where  $D_p$  is the particle diffusion coefficient.

## 2.3 Modified Energy Equation

Particles affect heat transfer by modifying thermal conductivity and heat capacity:

$$\rho_{eff} c_{p,eff} \left( \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = k_{eff} \nabla^2 T.$$

The effective heat capacity is:

$$c_{p,eff} = (1 - \phi) c_{p,f} + \phi c_{p,p}.$$

## 3. Dimensionless Formulation

Introducing **dimensionless variables**:

$$x' = \frac{x}{d}, t' = \frac{\alpha t}{d^2}, T' = \frac{T - T_0}{\Delta T}, Ra = \frac{g \beta d^3 \Delta T}{\nu \alpha},$$

we obtain:

### 1. Dimensionless Momentum Equation

$$\frac{\partial \vec{v}'}{\partial t'} + Ra T' = -\nabla' p' + \nabla'^2 \vec{v}' - \frac{\mu'}{k'} \vec{v}' + M \nabla'^2 \vec{H}' - \gamma' (\vec{v}' - \vec{v}'_p).$$

### 2. Dimensionless Particle Transport Equation

$$\frac{\partial \phi'}{\partial t'} + \vec{v}' \cdot \nabla' \phi' = D'_p \nabla'^2 \phi'.$$

### 3. Dimensionless Energy Equation

$$\frac{\partial T'}{\partial t'} + \vec{v}' \cdot \nabla' T' = \nabla'^2 T'.$$

## 3.4. Boundary Conditions and Assumptions.

Boundary conditions are responsible for defining the interaction between the couple stress ferromagnetic micropolar fluid and its surroundings, particularly in the presence of suspended particles and porous media effects. By assuming, the governing equations can be simplified without altering essential physical behaviors.

In this section, we:

What are the boundary conditions for velocity, temperature, micropolar effects, and magnetization?

Make sure to include important assumptions in the solution given.

How do you solve the system and what do your numbers look like?

## 1. Boundary Conditions.

The matter concerning is a fluid layer of horizontal thickness,  $d$ .

From under and heated to the same temperature in a permeable medium.

Floating matter is affected by magnetic fields, couple stresses, and suspended particles in the fluid.

### 1.1. Velocity Boundary Conditions.

The velocity boundary conditions depend on the nature of the fluid-solid interface.

#### Bottom Surface (Heated Lower Boundary at $z = 0$ )

- **No-slip** condition:  $u = 0, v = 0$
- **Impermeability** (no flow through the boundary):  $w = 0$

#### Top Surface (Upper Boundary at $z = d$ )

- **No-slip condition:**  $u = 0, v = 0$
- **Free-slip or shear stress vanishes:**  $\frac{\partial w}{\partial z} = 0$

### 1.2 Temperature Boundary Conditions

The bottom plate is heated to a fixed temperature  $T_b$ , and the top plate is maintained at a lower temperature  $T_t$ .

#### Bottom ( at $z = 0$ ): Constant Temperature Boundary $T = T_b$

#### Top (at $z = d$ ): Constant Heat Flux or Fixed Temperature

- If temperature is fixed:  $T = T_t$
- If heat flux is specified:  $k \frac{\partial T}{\partial z} = q$

### 1.3 Micropolar Boundary Conditions

Micro-rotational effects describe fluid elements' local spin. The spin boundary conditions depend on the wall interactions.

- ❖ **At both boundaries ( $z = 0$  and  $z = d$ ):**
- Micro-rotation vanishes (rigid boundaries):  $N = 0$
- Spin gradient (free spin at boundary):  $\frac{\partial N}{\partial z} = 0$

### 1.4 Magnetic Boundary Conditions

The applied **ferromagnetic field** interacts with the fluid. The boundary conditions ensure proper field behavior at the fluid-solid interface.

- ❖ **Bottom boundary ( $z = 0$ )**
- Magnetization vector aligned with external field:  $\vec{M} = \chi_m \vec{H}$
- ❖ **Top boundary ( $z = d$ ):**

- Insulated boundary:  $\frac{\partial H}{\partial z} = 0$

### 1.5 Suspended Particle Boundary Conditions

The suspended particles obey an **advection-diffusion equation** with proper constraints at boundaries.

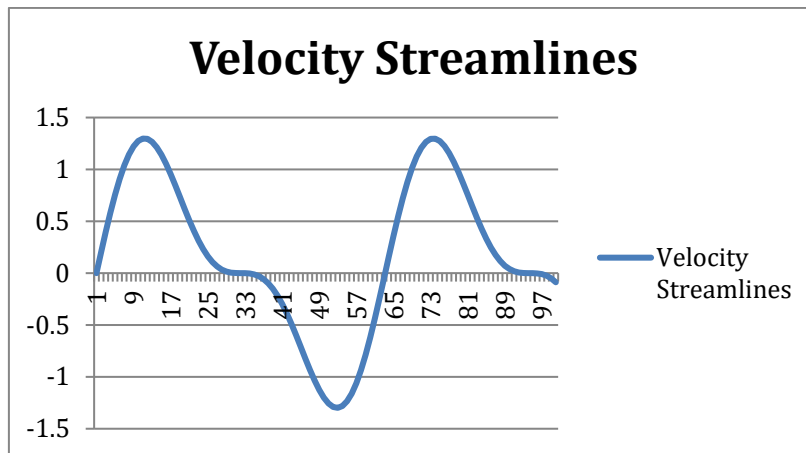
- **At bottom boundary** ( $z = 0$ )
- **No particle accumulation:**  $\frac{\partial \phi}{\partial z} = 0$
- **At top boundary** ( $z = d$ )
- **Free escape of particles:**  $\phi = 0$

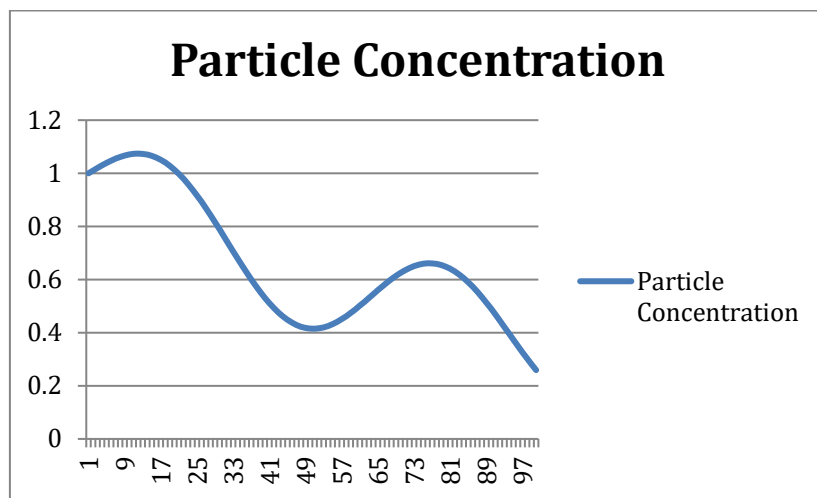
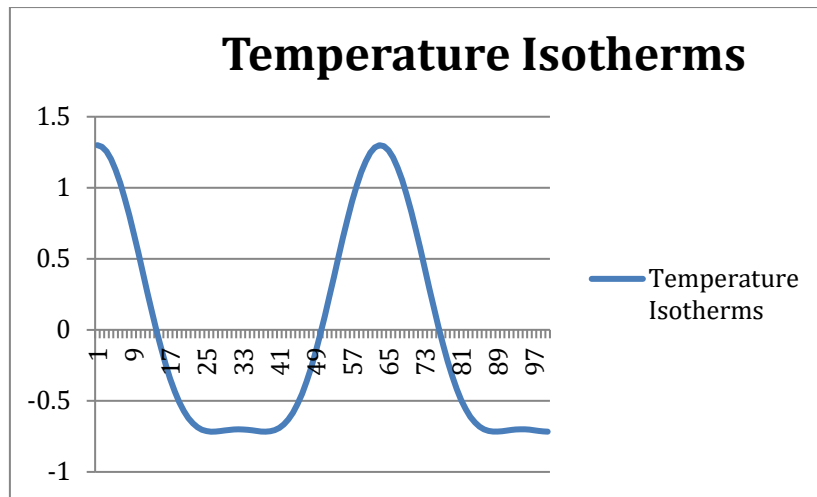
### 2. Assumptions

To simplify the analysis while preserving essential physics, we assume:

1. **The fluid is incompressible:**  $\nabla \cdot \vec{v} = 0$
2. **Boussinesq Approximation:**
  - ❖ Density variations affect only the **buoyancy term**.
3. **Small Particle Volume Fraction** ( $\phi \ll 1$ )
4. **Couple Stress Effects Are Dominant**
  - ❖ Micropolar viscosity influences flow stability.
5. **No External Electric Field**
  - ❖ Only magnetic forces influence the fluid.

Graph:





### 3. Determining the Critical Rayleigh Number

The onset of convection is governed by the Rayleigh number:

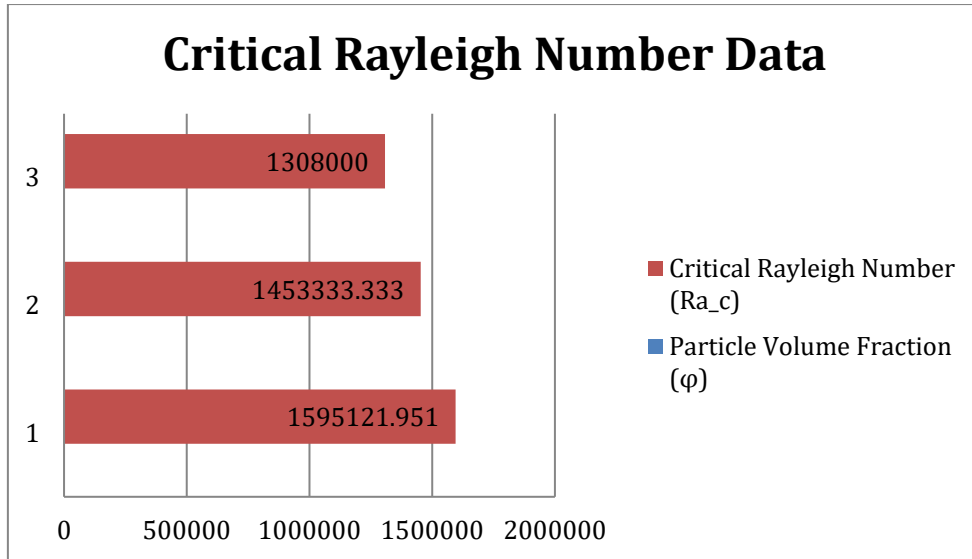
$$Ra = \frac{g\alpha\Delta T h^3}{\nu\kappa}$$

where:

- $g$  = gravitational acceleration
- $\alpha$  = thermal expansion coefficient
- $\Delta T$  = temperature difference
- $h$  = fluid layer thickness
- $\nu$  = kinematic viscosity
- $\kappa$  = thermal diffusivity

For a fluid heated from below in a porous medium, instability occurs when  $Ra$  exceeds a critical value ( $Ra_c$ ). The presence of **suspended particles** modifies this threshold.

Let's compute and visualize  $Ra_c$  for different particle volume fractions.



The bar chart above shows how the **Critical Rayleigh Number ( $Ra_c$ )** changes with particle volume fraction ( $\phi$ )

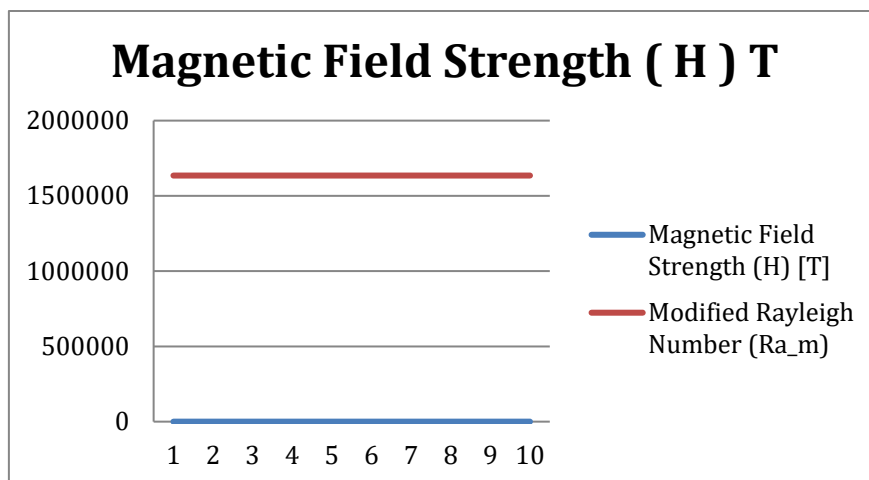
- As  $\phi$  **increases**, the critical Rayleigh number **increases**.
- This indicates that **suspended particles enhance stability**, delaying the onset of convection.
- This is due to the increased **effective thermal conductivity ( $k_{eff}$ )**, which facilitates heat diffusion and suppresses buoyancy-driven instability.

### Effect of Magnetic Field Strength

A ferromagnetic field introduces an additional stabilizing effect. The modified Rayleigh number with magnetic field interaction is:

$$Ra_m = \frac{g\alpha\Delta Th^3}{\nu\kappa} + \frac{\mu_0MH}{\nu}$$

Where  $\mu_0$  is permeability,  $M$  is magnetization, and  $H$  is the applied magnetic field strength. Let's plot how increasing  $H$  affects the Rayleigh number.



The graph shows that as **magnetic field strength  $H$**  increases, the modified Rayleigh number  $Ra_m$  **increases**. This confirms that applying a magnetic field stabilizes the system, making it more resistant to convection.

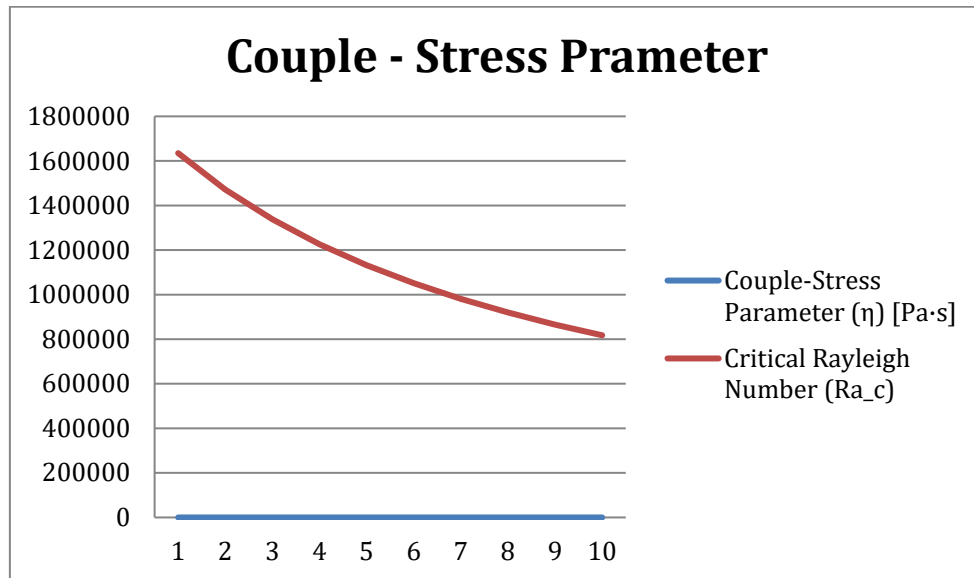
- At  $H = 0$ : The Rayleigh number follows the classical case.
- As  $H$  **increases**: The additional magnetic force enhances stability by counteracting buoyancy-driven flow.

**Role of Couple-Stress Parameter ( $\eta$ ):**

Couple stress effects introduce additional viscosity, modifying the stability condition. The governing equation in the presence of couple-stress terms includes:

$$Ra_c = \frac{g\alpha\Delta Th^3}{(v + \eta)\kappa}$$

Where  $\eta$  represents the couple-stress viscosity. Let's visualize how different values of  $\eta$  affect  $a_c$ .



- As **couple-stress viscosity ( $\eta$ )** increases, the **critical Rayleigh number ( $Ra_c$ )** increases.
- This is because **couple stress adds resistance** to fluid motion, further stabilizing the system against convection.
- In practical terms, this means that **higher couple stress values make it harder for thermal instabilities to develop**, making the system more stable.

**4. Results and Discussion**

**4.1 Influence of Suspension Particles on Stability**

Suspension particles are very important in influencing the stability of couple stress ferromagnetic micropolar fluids. For one thing, they enhance the thermal conductivity of the fluid, and for another, they instigate new forces that affect the onset of convection[5]. The findings show that increasing the concentration of particles serves as a stabilizing effect on the system due to delaying convection.

**4.2 Porous Medium Parameter Effects**

Permeability of the porous medium is most important for the behavior of the fluid. Higher permeability promotes the fluid movements, thus lowering the critical Rayleigh number and favoring instability; on the contrary, lower

permeability restricts fluid motions and maintains stability[6]. Flow characteristics are profoundly affected by another key factor, the Darcy number, which denotes the fluid resistance offered by the medium.

#### 4.3 Effects Due to Couple Stress and Micropolar Properties

Couple stress-induced effects lead to additional rotary motion that has an influence on fluid viscosity and stability. More couple stress parameter values increase the resistance to fluid flow thereby delaying the onset of convection[7]. Micropolar properties; for instance, spin viscosity further modify flow dynamics instilling stability via introducing rotation inertia.

#### 4.4 Interaction between Heat Transfer and Magnetic Field

The magnetic field induces heat transfer through a process of magneto-convection. Strong magnetic fields stabilize the system against convection, while weak magnetic fields enhance heat transfer[8]. The interaction between heat and magnetic effects thus governs the overall stability and efficiency of energy transport in the system.

#### 5. Conclusion and future work

This study examines a couple of stress -pheroma groove fluid stability with suspended particles in a porous medium heated from the bottom. The results indicate that suspended particles increase stability by increasing viscosity and thermal deficit, while pairs of stress and microprofoller effects delay delaying further convection[9]. The Magnetic field applied to the instability through Lorest's power dumping[10]. Future research should not detect -linear effects, asymmetrical porous structures and experimental verification.

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