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Av Ailability Analysis Of Hydro-Generator Power

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Abstract

Availability of the system is essential for the effective operation and main- tenance of repairable systems, especially in critical infrastructure such as Hydro- Electric Power Systems. The objective of this study is to apply the Markov Process Technique to a repairable system consisting of five components connected in parallel and evaluate the Mean Time Between Failures (MTBF), Mean Time To Repair (MTTR) and hence to calculate the Availability of the system. Finally the study ex- tends to analyse the Availability of Hydro-Generator Power System for the period of thirty years.

Keywords – Mean time between failures (MTBF), Mean time to repair (MTTR), Availability of the system.

Introduction

A Repairable system with five parallel components is a form of system that improves re- liability by allowing ongoing operation even if one or more components fail. In a parallel configuration, the system continues to work as long as at least one component is operational. This redundancy makes the system more durable and reduces full failure, which is especially important in applications that require continuous operation, such as power supply systems, communication networks, and safety-critical systems. The fundamental metrics for repairable systems is MTBF, MTTR and Availability of the System.

1 Repairable Systems

A Repairable system has a finite operational lifetime which can be restored to its original working condition. The availability of the system in such conditions typically increases over time due to maintenance or repairs. Strategies like preventive maintenance aim to reduce the frequency of failures, while corrective maintenance addresses issues as they arise. In this pa- per, the MTBF, MTTR and the Availability of the System have been derived for a repairable system with five parallel connected components.

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1.1 Notations and Descriptions

The notations and descriptions used in this paper are as follows:

Notations	Descriptions
λ_i	Failure rate of five components, i=1,2,3,4,5
μ_i	Repair rate of five components , i=1,2,3,4,5
$P_i\,(t)$	Probability of working of the system, at time 't', in ith*state, , i* = 1, 2, 3, ..., 22
T_i	Expected time to failure for state i, , i=1,2,3,...,21
MTBF	Mean Time Between Failure
MTTR	Mean Time to Repair
A(t)	Steady State Availability of the system

1.2 Transition Probability

Applying the Birth and Death process with λ failure rate and μ repair, we get the following Transition probability equation:

$$P_i\,(t + \Delta t) = (1 - (\lambda_i + \mu_j\,)\Delta t)P_j\,(t) + \mu_{j+1}\Delta t P_{j+1}(t) + \lambda_{j-1}\Delta t P_{j-1}(t)$$

Markov modelling uses state transition diagram to determine state sequences for the calculation of the probability of the system being in a particular state.

The transition state diagram and Transition Probability Matrix representing all states is outlined as follows:

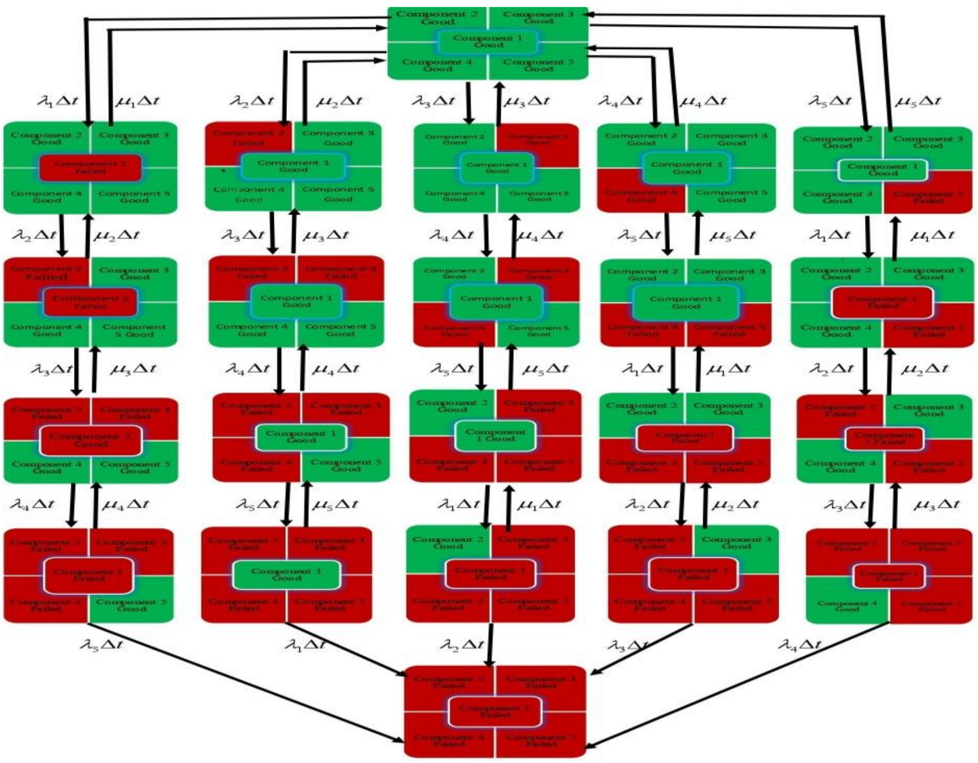

Figure 1: State Transition Diagram for Five Components

$$
\begin{pmatrix}
a_1 & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_1 & a_2 & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_2 & 0 & a_3 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & 0 & 0 & a_4 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_4 & 0 & 0 & 0 & a_5 & 0 & 0 & 0 & 0 & \mu_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_5 & 0 & 0 & 0 & 0 & a_6 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 & 0 & 0 & a_7 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & a_8 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & a_9 & 0 & 0 & 0 & 0 & \mu_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & 0 & a_{10} & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & a_{11} & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & a_{12} & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & a_{13} & 0 & 0 & 0 & 0 & \mu_5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & 0 & a_{14} & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & a_{15} & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & a_{16} & 0 & 0 & 0 & 0 & \mu_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & a_{17} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & 0 & a_{18} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & a_{19} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & a_{20} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & a_{21} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_5 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & 1
\end{pmatrix}
$$

where

$$a_1 = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)$$

$a_2 = -(\lambda_2 + \mu_1)$, $a_4 = -(\lambda_4 + \mu_3)$, $a_6 = -(\lambda_1 + \mu_5)$, $a_8 a_3 = -(\lambda_3 + \mu_2)$ $a_5 = -(\lambda_5 + \mu_4)$ $a_7 = -(\lambda_3 + \mu_2)$ $a_9 =$

$= -(\lambda_4 + \mu_3)$, $a_{10} = -(\lambda_1 + \mu_5)$, $a_{12} = -(\lambda_4 + \mu_3)$, $a_{14} -(\lambda_5 + \mu_4)$ $a_{11} = -(\lambda_2 + \mu_1)$ $a_{13} = -(\lambda_5 + \mu_4)$ $a_{15} = -(\lambda_5$

$= -(\lambda_1 + \mu_5)$, $+ \mu_4)$ $a_{17} = -\mu_4$

$a_{16} = -(\lambda_1 + \mu_5)$, $a_{19} = -\mu_1$

$a_{18} = -\mu_5$,

$a_{20} = -\mu_2$, $a_{21} = -\mu_3$

From the above Transition Probability Matrix we write the Probability Equation of the system in first state at $t + \Delta t$ as follows:

$$P_1(t+\Delta t) = (1-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)\Delta t)P_1(t)+\mu_1\Delta tP_2(t)+\mu_2\Delta tP_3(t)+\mu_3\Delta tP_4(t)+\mu_4\Delta tP_5(t)+\mu_5\Delta tP_6(t)$$

$$P_1(t+\Delta t)-P_1(t) = -(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)\Delta tP_1(t)+\mu_1\Delta tP_2(t)+\mu_2\Delta tP_3(t)+\mu_3\Delta tP_4(t)+\mu_4\Delta tP_5(t)+\mu_5\Delta tP_6(t)$$

Taking Limit $\Delta t \to 0$, we get the following differential equation:

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)P_1(t) + \mu_1 P_2(t) + \mu_2 P_3(t) + \mu_3 P_4(t) + \mu_4 P_5(t) + \mu_5 P_6(t)$$

Integrating and applying the boundary conditions, $P_1(t) = 1$, $P_2(t) = 0$, $P_3(t) = 0$, ..., $P_{21}(t) = 0$, $P_{22}(t) = 1$

$$1 = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)T_1 + \mu_1 T_2 + \mu_2 T_3 + \mu_3 T_4 + \mu_4 T_5 + \mu_5 T_6$$

Probability of the system in the second state:

$$P_2(t + \Delta t) = \lambda_1 \Delta t P_1(t) + (1 - (\lambda_2 + \mu_1))\Delta t P_2(t) + \mu_2 \Delta t P_7(t)$$

Taking Limit $\Delta t \to 0$ and solving the differential equation, we get

$$0 = \lambda_1 T_1 - (\lambda_2 + \mu_1)T_2 + \mu_2 T_7$$

$$T_2 = \frac{\lambda_1}{(\lambda_2 + \mu_1)} T_1 + \frac{\mu_2}{(\lambda_2 + \mu_1)} T_7 \quad (1)$$

Similarly from the probability of the system in state 3, 4, 5, ... 21 we derive the following equations:

$$T_3 = \frac{\lambda_2}{(\lambda_3 + \mu_2)} T_1 + \frac{\mu_3}{(\lambda_3 + \mu_2)} T_8 \quad (2)$$

$$T_4 = \frac{\lambda_3}{(\lambda_4 + \mu_3)} T_1 + \frac{\mu_4}{(\lambda_4 + \mu_3)} T_9 \quad (3)$$

$$T_5 = \frac{\lambda_4}{(\lambda_5 + \mu_4)} T_1 + \frac{\mu_5}{(\lambda_5 + \mu_4)} T_{10} \quad (4)$$

$$T_6 = \frac{\lambda_5}{(\lambda_1 + \mu_5)} T_1 + \frac{\mu_1}{(\lambda_1 + \mu_5)} T_{11} \quad (5)$$

$$T_7 = \frac{\lambda_2}{(\lambda_3 + \mu_2)} T_2 + \frac{\mu_3}{(\lambda_3 + \mu_2)} T_{12} \quad (6)$$

$$T_8 = \frac{\lambda_3}{(\lambda_4 + \mu_3)} T_3 + \frac{\mu_4}{(\lambda_4 + \mu_3)} T_{13} \quad (7)$$

$$T_9 = \frac{\lambda_4}{(\lambda_5 + \mu_4)} T_4 + \frac{\mu_5}{(\lambda_5 + \mu_4)} T_{14} \quad (8)$$

$$T_{10} = \frac{\lambda_5}{(\lambda_1 + \mu_5)} T_5 + \frac{\mu_1}{(\lambda_1 + \mu_5)} T_{15} \quad (9)$$

$$T_{11} = \frac{\lambda_1}{(\lambda_2 + \mu_1)} T_6 + \frac{\mu_2}{(\lambda_2 + \mu_1)} T_{16} \quad (10)$$

$$T_{12} = \frac{\lambda_3}{(\lambda_4 + \mu_3)} T_7 + \frac{\mu_4}{(\lambda_4 + \mu_3)} T_{17} \quad (11)$$

$$T_{13} = \frac{\lambda_4}{(\lambda_5 + \mu_4)} T_8 + \frac{\mu_5}{(\lambda_5 + \mu_4)} T_{18} \quad (12)$$

$$T_{14} = \frac{\lambda_5}{(\lambda_1 + \mu_5)} T_9 + \frac{\mu_1}{(\lambda_1 + \mu_5)} T_1 \quad (13)$$

$$T_{15} = \frac{\lambda_1}{(\lambda_2 + \mu_1)} T_{10} + \frac{\mu_2}{(\lambda_2 + \mu_1)} T_{20} \quad (14)$$

$$T_{16} = \frac{\lambda_2}{(\lambda_3 + \mu_2)} T_{11} + \frac{\mu_3}{(\lambda_3 + \mu_2)} T_{21} \quad (15)$$

$$T_{17} = \frac{\lambda_4}{\lambda_5 + \mu_4} T_{12} \quad (16)$$

$$T_{18} = \frac{\lambda_5}{\lambda_1 + \mu_5} T_{13} \quad (17)$$

$$T_{19} = \frac{\lambda_1}{\lambda_2 + \mu_1} T_{14} \quad (18)$$

$$T_{20} = \frac{\lambda_2}{\lambda_3 + \mu_2} T_{15} \quad (19)$$

$$T_{21} = \frac{\lambda_3}{\lambda_4 + \mu_3} T_{16} \quad (20)$$

Probability of the system in 22nd state is

$P_{22}(t + \Delta t) = P_{22}(t) + \lambda_5 \Delta t P_{17}(t) + \lambda_1 \Delta t P_{18}(t) + \lambda_2 \Delta t P_{19}(t) + \lambda_3 \Delta t P_{20}(t) + \lambda_4 \Delta t P_{21}(t)$ Integrating and applying the boundary conditions (1), we get,

$$1 = \lambda_5 T_{17} + \lambda_1 T_{18} + \lambda_2 T_{19} + \lambda_3 T_{20} + \lambda_4 T_{21} \quad (21)$$

Solving the above equations, we get

$$T_2 = \frac{\lambda_1\lambda_3\lambda_4\lambda_5 + \lambda_1\lambda_4\lambda_5\mu_2 + \lambda_1\lambda_5\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4}{\lambda_2\lambda_3\lambda_4\lambda_5 + \lambda_3\lambda_4\lambda_5\mu_1 + \lambda_4\lambda_5\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4} \cdot T_1$$

$$T_3 = \frac{\lambda_2\lambda_4\lambda_5\lambda_1 + \lambda_2\lambda_5\lambda_1\mu_3 + \lambda_2\lambda_1\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5}{\lambda_3\lambda_4\lambda_5\lambda_1 + \lambda_4\lambda_5\lambda_1\mu_2 + \lambda_5\lambda_1\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4 + \mu_2\mu_3\mu_4\mu_5} \cdot T_1$$

$$T_4 = \frac{\lambda_3\lambda_5\lambda_1\lambda_2 + \lambda_3\lambda_1\lambda_2\mu_4 + \lambda_3\lambda_2\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1}{\lambda_4\lambda_5\lambda_1\lambda_2 + \lambda_5\lambda_1\lambda_2\mu_3 + \lambda_1\lambda_2\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5 + \mu_3\mu_4\mu_5\mu_1} \cdot T_1$$

$$T_5 = \frac{\lambda_4\lambda_1\lambda_2\lambda_3 + \lambda_4\lambda_2\lambda_3\mu_5 + \lambda_4\lambda_3\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2}{\lambda_5\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_3\mu_4 + \lambda_2\lambda_3\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1 + \mu_4\mu_5\mu_1\mu_2} \cdot T_1$$

$$T_6 = \frac{\lambda_5\lambda_2\lambda_3\lambda_4 + \lambda_5\lambda_3\lambda_4\mu_1 + \lambda_5\lambda_4\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3}{\lambda_1\lambda_2\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\mu_5 + \lambda_3\lambda_4\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2 + \mu_5\mu_1\mu_2\mu_3} \cdot T_1$$

$$T_7 = \frac{\lambda_2(\lambda_4\lambda_5 + \mu_3\mu_4 + \lambda_5\mu_3)}{\lambda_3\lambda_4\lambda_5 + \lambda_4\lambda_5\mu_2 + \lambda_5\mu_2\mu_3 + \mu_2\mu_3\mu_4} \cdot \frac{\lambda_1\lambda_3\lambda_4\lambda_5 + \lambda_1\lambda_4\lambda_5\mu_2 + \lambda_1\lambda_5\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4}{\lambda_2\lambda_3\lambda_4\lambda_5 + \lambda_3\lambda_4\lambda_5\mu_1 + \lambda_4\lambda_5\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4} \cdot T_1$$

$$T_8 = \frac{\lambda_3(\lambda_5\lambda_1 + \mu_4\mu_5 + \lambda_1\mu_4)}{\lambda_4\lambda_5\lambda_1 + \lambda_5\lambda_1\mu_3 + \lambda_1\mu_3\mu_4 + \mu_3\mu_4\mu_5} \cdot \frac{\lambda_2\lambda_4\lambda_5\lambda_1 + \lambda_2\lambda_5\lambda_1\mu_3 + \lambda_2\lambda_1\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5}{\lambda_3\lambda_4\lambda_5\lambda_1 + \lambda_4\lambda_5\lambda_1\mu_2 + \lambda_5\lambda_1\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4 + \mu_2\mu_3\mu_4\mu_5} \cdot T_1$$

$$T_9 = \frac{\lambda_4(\lambda_1\lambda_2 + \mu_5\mu_1 + \lambda_2\mu_5)}{\lambda_5\lambda_1\lambda_2 + \lambda_1\lambda_2\mu_4 + \lambda_2\mu_4\mu_5 + \mu_4\mu_5\mu_1} \cdot \frac{\lambda_3\lambda_5\lambda_1\lambda_2 + \lambda_3\lambda_1\lambda_2\mu_4 + \lambda_3\lambda_2\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1}{\lambda_4\lambda_5\lambda_1\lambda_2 + \lambda_5\lambda_1\lambda_2\mu_3 + \lambda_1\lambda_2\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5 + \mu_3\mu_4\mu_5\mu_1} \cdot T_1$$

$$T_{10} = \frac{\lambda_5(\lambda_2\lambda_3 + \mu_1\mu_2 + \lambda_3\mu_1)}{\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\mu_5 + \lambda_3\mu_5\mu_1 + \mu_5\mu_1\mu_2} \cdot \frac{\lambda_4\lambda_1\lambda_2\lambda_3 + \lambda_4\lambda_2\lambda_3\mu_5 + \lambda_4\lambda_3\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2}{\lambda_5\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_3\mu_4 + \lambda_2\lambda_3\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1 + \mu_4\mu_5\mu_1\mu_2} \cdot T_1$$

$$T_{11} = \frac{\lambda_1(\lambda_3\lambda_4 + \mu_2\mu_3 + \lambda_4\mu_2)}{\lambda_2\lambda_3\lambda_4 + \lambda_3\lambda_4\mu_1 + \lambda_4\mu_5\mu_2 + \mu_1\mu_2\mu_3} \cdot \frac{\lambda_5\lambda_2\lambda_3\lambda_4 + \lambda_5\lambda_3\lambda_4\mu_1 + \lambda_5\lambda_4\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3}{\lambda_1\lambda_2\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\mu_5 + \lambda_3\lambda_4\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2 + \mu_5\mu_1\mu_2\mu_3} \cdot T_1$$

$$T_{12} = \frac{\lambda_3(\lambda_5 + \mu_4)}{\lambda_4\lambda_5 + \lambda_4\mu_3 + \mu_3\mu_4} \cdot \frac{\lambda_2(\lambda_4\lambda_5 + \mu_3\mu_4 + \lambda_5\mu_3)}{\lambda_3\lambda_4\lambda_5 + \lambda_4\lambda_5\mu_2 + \lambda_5\mu_2\mu_3 + \mu_2\mu_3\mu_4} \cdot \frac{\lambda_1\lambda_3\lambda_4\lambda_5 + \lambda_1\lambda_4\lambda_5\mu_2 + \lambda_1\lambda_5\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4}{\lambda_2\lambda_3\lambda_4\lambda_5 + \lambda_3\lambda_4\lambda_5\mu_1 + \lambda_4\lambda_5\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4} \cdot T_1$$

$$T_{13} = \frac{\lambda_4(\lambda_1 + \mu_5)}{\lambda_5\lambda_1 + \lambda_4\mu_4 + \mu_4\mu_5} \cdot \frac{\lambda_3(\lambda_5\lambda_1 + \mu_4\mu_5 + \lambda_1\mu_4)}{\lambda_4\lambda_5\lambda_1 + \lambda_5\lambda_1\mu_3 + \lambda_1\mu_3\mu_4 + \mu_3\mu_4\mu_5} \cdot \frac{\lambda_2\lambda_4\lambda_5\lambda_1 + \lambda_2\lambda_5\lambda_1\mu_3 + \lambda_2\lambda_1\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5}{\lambda_3\lambda_4\lambda_5\lambda_1 + \lambda_4\lambda_5\lambda_1\mu_2 + \lambda_5\lambda_1\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4 + \mu_2\mu_3\mu_4\mu_5} \cdot T_1$$

$$T_{14} = \frac{\lambda_5(\lambda_2 + \mu_1)}{\lambda_1\lambda_2 + \lambda_5\mu_5 + \mu_5\mu_1} \cdot \frac{\lambda_4(\lambda_1\lambda_2 + \mu_5\mu_1 + \lambda_2\mu_5)}{\lambda_5\lambda_1\lambda_2 + \lambda_1\lambda_2\mu_4 + \lambda_2\mu_4\mu_5 + \mu_4\mu_5\mu_1} \cdot \frac{\lambda_3\lambda_5\lambda_1\lambda_2 + \lambda_3\lambda_1\lambda_2\mu_4 + \lambda_3\lambda_2\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1}{\lambda_4\lambda_5\lambda_1\lambda_2 + \lambda_5\lambda_1\lambda_2\mu_3 + \lambda_1\lambda_2\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5 + \mu_3\mu_4\mu_5\mu_1} \cdot T_1$$

$$T_{15} = \frac{\lambda_1(\lambda_3 + \mu_2)}{\lambda_2\lambda_3 + \lambda_1\mu_1 + \mu_5\mu_2} \cdot \frac{\lambda_5(\lambda_2\lambda_3 + \mu_1\mu_2 + \lambda_3\mu_1)}{\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\mu_5 + \lambda_3\mu_5\mu_1 + \mu_5\mu_1\mu_2} \cdot \frac{\lambda_4\lambda_1\lambda_2\lambda_3 + \lambda_4\lambda_2\lambda_3\mu_5 + \lambda_4\lambda_3\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2}{\lambda_5\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_3\mu_4 + \lambda_2\lambda_3\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1 + \mu_4\mu_5\mu_1\mu_2} \cdot T_1$$

$$T_{16} = \frac{\lambda_2(\lambda_4 + \mu_3)}{\lambda_3\lambda_4 + \lambda_2\mu_2 + \mu_1\mu_3} \cdot \frac{\lambda_1(\lambda_3\lambda_4 + \mu_2\mu_3 + \lambda_4\mu_2)}{\lambda_2\lambda_3\lambda_4 + \lambda_3\lambda_4\mu_1 + \lambda_4\mu_5\mu_2 + \mu_1\mu_2\mu_3} \cdot$$

$$\frac{\lambda_5\lambda_2\lambda_3\lambda_4 + \lambda_5\lambda_3\lambda_4\mu_1 + \lambda_5\lambda_4\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3}{\lambda_1\lambda_2\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\mu_5 + \lambda_3\lambda_4\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2 + \mu_5\mu_1\mu_2\mu_3} \cdot T_1$$

$$T_{17} = \frac{\lambda_4}{\lambda_5 + \mu_4} \cdot \frac{\lambda_3(\lambda_5 + \mu_4)}{\lambda_4\lambda_5 + \lambda_4\mu_3 + \mu_3\mu_4} \cdot \frac{\lambda_2(\lambda_4\lambda_5 + \mu_3\mu_4 + \lambda_5\mu_3)}{\lambda_3\lambda_4\lambda_5 + \lambda_4\lambda_5\mu_2 + \lambda_5\mu_2\mu_3 + \mu_2\mu_3\mu_4} \cdot \frac{\lambda_1\lambda_3\lambda_4\lambda_5 + \lambda_1\lambda_4\lambda_5\mu_2 + \lambda_1\lambda_5\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4}{\lambda_2\lambda_3\lambda_4\lambda_5 + \lambda_3\lambda_4\lambda_5\mu_1 + \lambda_4\lambda_5\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4} \cdot T_1$$

$$T_{18} = \frac{\lambda_5}{\lambda_1 + \mu_5} \cdot \frac{\lambda_4(\lambda_1 + \mu_5)}{\lambda_5\lambda_1 + \lambda_4\mu_4 + \mu_4\mu_5} \cdot \frac{\lambda_3(\lambda_5\lambda_1 + \mu_4\mu_5 + \lambda_1\mu_4)}{\lambda_4\lambda_5\lambda_1 + \lambda_5\lambda_1\mu_3 + \lambda_1\mu_3\mu_4 + \mu_3\mu_4\mu_5} \cdot \frac{\lambda_2\lambda_4\lambda_5\lambda_1 + \lambda_2\lambda_5\lambda_1\mu_3 + \lambda_2\lambda_1\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5}{\lambda_3\lambda_4\lambda_5\lambda_1 + \lambda_4\lambda_5\lambda_1\mu_2 + \lambda_5\lambda_1\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4 + \mu_2\mu_3\mu_4\mu_5} \cdot T_1$$

$$T_{19} = \frac{\lambda_1}{\lambda_2 + \mu_1} \cdot \frac{\lambda_5(\lambda_2 + \mu_1)}{\lambda_1\lambda_2 + \lambda_5\mu_5 + \mu_5\mu_1} \cdot \frac{\lambda_4(\lambda_1\lambda_2 + \mu_5\mu_1 + \lambda_2\mu_5)}{\lambda_5\lambda_1\lambda_2 + \lambda_1\lambda_2\mu_4 + \lambda_2\mu_4\mu_5 + \mu_4\mu_5\mu_1} \cdot \frac{\lambda_3\lambda_5\lambda_1\lambda_2 + \lambda_3\lambda_1\lambda_2\mu_4 + \lambda_3\lambda_2\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1}{\lambda_4\lambda_5\lambda_1\lambda_2 + \lambda_5\lambda_1\lambda_2\mu_3 + \lambda_1\lambda_2\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5 + \mu_3\mu_4\mu_5\mu_1} \cdot T_1$$

$$T_{20} = \frac{\lambda_2}{\lambda_3 + \mu_2} \cdot \frac{\lambda_1(\lambda_3 + \mu_2)}{\lambda_2\lambda_3 + \lambda_1\mu_1 + \mu_5\mu_2} \cdot \frac{\lambda_5(\lambda_2\lambda_3 + \mu_1\mu_2 + \lambda_3\mu_1)}{\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\mu_5 + \lambda_3\mu_5\mu_1 + \mu_5\mu_1\mu_2} \cdot \frac{\lambda_4\lambda_1\lambda_2\lambda_3 + \lambda_4\lambda_2\lambda_3\mu_5 + \lambda_4\lambda_3\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2}{\lambda_5\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_3\mu_4 + \lambda_2\lambda_3\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1 + \mu_4\mu_5\mu_1\mu_2} \cdot T_1$$

$$T_{21} = \frac{\lambda_3}{\lambda_4 + \mu_3} \cdot \frac{\lambda_2(\lambda_4 + \mu_3)}{\lambda_3\lambda_4 + \lambda_2\mu_2 + \mu_1\mu_3} \cdot \frac{\lambda_1(\lambda_3\lambda_4 + \mu_2\mu_3 + \lambda_4\mu_2)}{\lambda_2\lambda_3\lambda_4 + \lambda_3\lambda_4\mu_1 + \lambda_4\mu_5\mu_2 + \mu_1\mu_2\mu_3} \cdot \frac{\lambda_5\lambda_2\lambda_3\lambda_4 + \lambda_5\lambda_3\lambda_4\mu_1 + \lambda_5\lambda_4\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3}{\lambda_1\lambda_2\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4\mu_5 + \lambda_3\lambda_4\mu_5\mu_1 + \lambda_4\mu_5\mu_1\mu_2 + \mu_5\mu_1\mu_2\mu_3} \cdot T_1$$

Substituting the values of T_{17}, T_{18}, T_{19}, T_{20}, T_{21} in equation (22) we get,

$$1 = \lambda_5 \cdot \frac{\lambda_4}{\lambda_5 + \mu_4} \cdot \frac{\lambda_3(\lambda_5 + \mu_4)}{\lambda_4\lambda_5 + \lambda_4\mu_3 + \mu_3\mu_4} \cdot \frac{\lambda_2(\lambda_4\lambda_5 + \mu_3\mu_4 + \lambda_5\mu_3)}{\lambda_3\lambda_4\lambda_5 + \lambda_4\lambda_5\mu_2 + \lambda_5\mu_2\mu_3 + \mu_2\mu_3\mu_4} \cdot \frac{\lambda_1\lambda_3\lambda_4\lambda_5 + \lambda_1\lambda_4\lambda_5\mu_2 + \lambda_1\lambda_5\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4}{\lambda_2\lambda_3\lambda_4\lambda_5 + \lambda_3\lambda_4\lambda_5\mu_1 + \lambda_4\lambda_5\mu_1\mu_2 + \lambda_5\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4} \cdot T_1$$

$$+\lambda_1 \cdot \frac{\lambda_5}{\lambda_1 + \mu_5} \cdot \frac{\lambda_4(\lambda_1 + \mu_5)}{\lambda_5\lambda_1 + \lambda_4\mu_4 + \mu_4\mu_5} \cdot \frac{\lambda_3(\lambda_5\lambda_1 + \mu_4\mu_5 + \lambda_1\mu_4)}{\lambda_4\lambda_5\lambda_1 + \lambda_5\lambda_1\mu_3 + \lambda_1\mu_3\mu_4 + \mu_3\mu_4\mu_5} \cdot \frac{\lambda_2\lambda_4\lambda_5\lambda_1 + \lambda_2\lambda_5\lambda_1\mu_3 + \lambda_2\lambda_1\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5}{\lambda_3\lambda_4\lambda_5\lambda_1 + \lambda_4\lambda_5\lambda_1\mu_2 + \lambda_5\lambda_1\mu_2\mu_3 + \lambda_1\mu_2\mu_3\mu_4 + \mu_2\mu_3\mu_4\mu_5} \cdot T_1$$

$$+\lambda_2 \cdot \frac{\lambda_1}{\lambda_2 + \mu_1} \cdot \lambda_5(\lambda_2 + \mu_1)$$

$\lambda_1\lambda_2 + \lambda_5\mu_5 + \mu_5\mu_1 \quad \frac{\lambda_4(\lambda_1\lambda_2 + \mu_5\mu_1 + \lambda_2\mu_5)}{\lambda_5\lambda_1\lambda_2 + \lambda_1\lambda_2\mu_4 + \lambda_2\mu_4\mu_5 + \mu_4\mu_5\mu_1}$

$$\frac{\lambda_3\lambda_5\lambda_1\lambda_2 + \lambda_3\lambda_1\lambda_2\mu_4 + \lambda_3\lambda_2\mu_4\mu_5 + \lambda_3\mu_4\mu_5\mu_1}{\lambda_4\lambda_5\lambda_1\lambda_2 + \lambda_5\lambda_1\lambda_2\mu_3 + \lambda_1\lambda_2\mu_3\mu_4 + \lambda_2\mu_3\mu_4\mu_5 + \mu_3\mu_4\mu_5\mu_1}$$

λ

λ5λ2λ3λ4 + λ5λ3λ4μ1 +

λ5λ4μ1μ2 + λ5μ1μ2μ3 5 2 1

+λ λ2 λ3 λ1λ2λ3λ4 + λ2λ3λ4μ5 + μ

+ μ2 λ3λ4μ5μ1 + λ4μ5μ1μ2 +

μ5μ1μ2μ3 5 2 3 1 2 3 4

λ1(λ3 + μ2) μ μ μ □

λ2λ3 + λ1μ1 + μ5μ2 . T1

□ . 2 .3. 4 5

λ5(λ2λ3 + μ1μ2 + λ3μ1) □□ λ

3 4 5 1

λ1λ2λ3 + λ2λ3μ5 + . λ4 . . λ3(λ5+μ4)

λ3μ5μ1 + μ5μ1μ2 . . .4. 5 1 2

λ2(λ4λ5+μ3μ4

λ4λ1λ2λ3 + λ4λ2λ3μ5 + +λ5μ3) . 5 1 2 3

λ4λ3μ5μ1 + λ4μ5μ1μ2

λ5λ1λ2λ3 + λ1λ2λ3μ4 + □□−1 1 2 3 4. □

λ2λ3μ4μ5 + λ3μ4μ5μ1 +

μ4μ5μ1μ2 □□□ □□□□

. T1 5 λ5+μ4

λ4λ5+λ4μ3+μ

+λ λ3 λ4 3μ4 λ1+μ5 λ5λ1+λ4μ4+μ4μ5

+ μ3 λ3λ4λ5+λ4λ5μ λ

2+λ5μ2μ3+μ2μ3μ4

λ2(λ4 + μ3) λ4λ5λ1+λ5λ1μ3+λ1μ3μ4+μ3μ4μ5

λ3λ4 + λ2μ2 + μ1μ3 □□□

□□□

λ1(λ3λ4 + μ2μ3 + λ4μ2) 1 3

□ λ λ λ □□□□

λ2λ3λ4 + λ3λ4μ1 +

λ4μ5μ2 + μ1μ2μ3 4 5 1 4 2λ4λ5λ1+λ2λ5λ1μ3+λ2λ1μ3μ4+λ2μ3μ4μ5 λ3λ4λ5λ1+λ4λ5λ1μ2+λ5λ

1μ2μ3+λ1μ2μ3μ4+μ2μ3μ

4μ5

. □□□□□

T1 = □□□□

λ2+μ1

λ1λ2+λ5μ5+μ5

μ1

λ

λ5λ1λ2+λ1λ2μ4+λ2μ4μ5

+μ4μ5μ1

□□□□

□□□□

3λ5λ1λ2+λ3λ1λ2μ4+λ3λ2

μ4μ5+λ3μ4μ5μ1

λ4λ5λ1λ2+λ5λ1λ2μ3+λ1λ

2μ3μ4+λ2μ3μ4μ5+μ3μ4μ

5μ1

. □□□□□□

□□□□□

λ3+μ2

λ2λ3+λ1μ1+μ5

μ2

λ

λ1λ2λ3+λ2λ3μ5+λ3μ5μ1

+μ5μ1μ2

□□□□

□□□□

4λ1λ2λ3+λ4λ2λ3μ5+λ4λ

3μ5μ1+λ4μ5μ1μ2

λ5λ1λ2λ3+λ1λ2λ3μ4+λ2

λ3μ4μ5+λ3μ4μ5μ1+μ4μ

5μ1μ2

. □□□□□□

+λ4

□

λ3

λ4+μ3

λ2(λ4+μ3)

λ3λ4+λ2μ2+μ1μ3

λ1(λ3λ4+μ2μ3

+λ4μ2)

λ2λ3λ4+λ3λ4μ1+λ4μ5μ2

+μ1μ2μ3

□□□

MTBF is the sum of the expected time for failure from state 1 to state 21

MT BF = T1 + T2 + T3 + T4 + T5 + T6 + T7 + T8 + T9 + T10 + T11 + T12 + T13 + T14 + T15 + T16 + T17 + T18 +

T19 + T20 + T21

Let us assume that the system have equal failure rate and repair rate.

λ1 = λ2 = λ3 = λ4 = λ5 = λ and μ1 = μ2 = μ3 = μ4 = μ5 = μ

21λ4 + 11λ3μ + 11λ2μ2 + 11λμ3 + μ4

MT BF =

5λ5

MTTR is the average time required to repair a component which is under failure condition.

1

MT T R = μ1 + μ2 + μ3 + μ4 + μ5

For equal repair rate,

1

MT T R = 5μ

The Availability of the system after some period of time is

A(t) =

MT BF MT BF + MT T R =

μ .21λ4 + 11λ3μ + 11λ2μ2 + 11λμ3 + μ4Σ

μ .21λ4 + 11λ3μ + 11λ2μ2 + 11λμ3 + μ4Σ + λ5

1.3 Power systems of Hydro Generator

Hydro-Generators are important components of Hydro-Electric Power Plants that trans- form the kinetic energy of flowing or falling water into electricity. They work by forcing water through turbines, which spin a rotor within the generator, creating an electromagnetic field that produces electric energy. This technique is highly efficient, transforming up to 90 percent- age of water's energy intopower.

In this study, a power plant consisting of five Hydro Generators have been illustrated. We look at the performance of Hydro-Generators over the period of 30 years. The failure and repair rates of all five generators are assumed to be equal. These rates are analyzed at 3-year intervals, resulting in ten sets of data. For each 3-year period, metrics like Mean Time Between Failures (MTBF), Mean Time to Repair (MTTR), and System Availability have been calculated. Finally, the Availability of Hydro-Generator Power Systems after 30 years isestimated.

S. No.	Failure Rate λ	Repair Rate μ	MTBF	MTTR	Availability A(t→ ∞)
1	0.20	0.20	55.00000	1.00000	0.98214
2	0.22	0.18	39.85141	1.11111	0.97287
3	0.24	0.16	30.56584	1.25000	0.96071
4	0.26	0.14	24.54910	1.42857	0.94501
5	0.28	0.12	20.45308	1.66667	0.92465
6	0.30	0.10	17.53909	2.00000	0.89764
7	0.32	0.08	15.38330	2.50000	0.86020
8	0.34	0.06	13.73244	3.33333	0.80467
9	0.36	0.04	12.42959	5.00000	0.71313
10	0.38	0.02	11.37422	10.0000	0.53214

Table 1: MTBF, MTTR and System Availability of Hydro-Generators

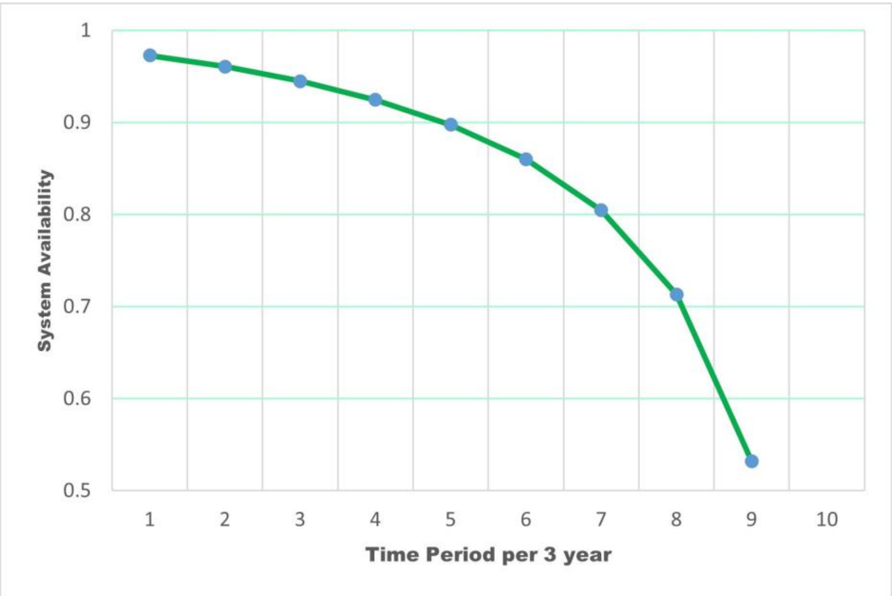

Figure 2: Availability of the System

From the above Figure, we can observe that the Mean Time Between Failures (MTBF) has been decreasing, while the Mean Time to Repair (MTTR) is increasing. Moreover, during the initial period, the system's availability was 98 percentage, but as the years progress, the avail- ability gradually decreases. After thirty years, the system's availability declines to 53 per- centage. This study provides valuable insights into the probability of the Hydrogenerator's efficiency over a 30-year period.

Conclusion

The Availability analysis of a Repairable system with five parallel components, using Markov processes, provided important information about the system's performance, includ- ing critical characteristics such as Mean Time Between Failures (MTBF), Mean Time to Repair (MTTR) and System Availability. These measures emphasize the system's dependability and serve as a foundation for optimizing maintenance efforts. Furthermore, the application of these technologies to a Hydro-Electric Generating Power System demonstrates their importance to essential infrastructure, ensuring long-term and efficient functioning.

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