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A Comparative Study of Exponential and Weibull Distributions to Improve Treatment Strategies

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Abstract

Reliability analysis, commonly used in engineering, can be applied in human pathology to understand disease progression and treatment outcomes. The **exponential distribution**, with a constant failure rate, suits conditions with steady risk, while the **Weibull distribution**, which allows varying rates, is better for modelling age-related diseases or changing treatment effects. Using these models helps build accurate predictions, improve treatment strategies, and enhance patient care over a lifetime.

Keywords: Repair Rate, Reliability(Ʀ), Convolution Integral, Weibull and Exponential Distribution.

I. INTRODUCTION

Human pathology, also referred to as general pathology, is the scientific study of diseases, focusing on their causes (etiology), development (pathogenesis), and effects on human health. Pathologists examine disease mechanisms to aid in accurate diagnosis and understand how illnesses progress in both individuals and populations. This discipline is essential to healthcare, medical research, and public health, contributing significantly to disease prevention, diagnosis, treatment, and overall human well-being. Despite considerable advancements, human pathology faces challenges such as the rapid mutation of pathogens like viruses and bacteria, which complicates diagnosis and treatment.

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Additionally, environmental factors such as climate, pollution, and lifestyle can influence disease development, making predictions difficult. Limited access to advanced diagnostic tools due to resource constraints also creates disparities in healthcare delivery. Addressing these challenges requires better disease surveillance, broader access to technology, and ongoing research efforts.

Ensuring the reliability of human pathology is critical for effective disease control and better health outcomes. The integration of modern molecular methods and artificial intelligence with traditional diagnostic practices significantly improves accuracy and efficiency in disease detection. Investing in pathology research and ensuring widespread access to reliable diagnostic tools enhances global health security and reduces disease burdens. Historical figures like Rudolf Virchow, the father of modern pathology, and Sir Ronald Ross, who made pivotal contributions to malaria research in India, have shaped the field. In technical terms, reliability refers to the probability that a system or component will function as intended over a specific time under defined conditions. This is often modeled using the formula $R(t) = e^{-\lambda t}$, where λ represents the constant failure rate. In human pathology, this model helps assess the durability of diagnostic tests and treatment effectiveness over time, considering factors such as drug resistance and the risk of inaccurate test results.

II. BASIC DEFINITIONS

2.1 Reliability:

Reliability is the ability of a system, component or process to consistently perform its intended function without failure over a specify period under normal operating conditions. It reflects the dependability, stability and trustworthiness of something ensuring that it meets performance expectations repeatedly and predictably.

Reliability is defined as,

$$R(t) = \int_t^{\infty} f(x)dx$$

R(t) is usually called reliability function and F(x) is the Probability density function of the system failure at time t.

2.2 Exponential Distribution:

The Exponential distribution is a special case of Weibull distribution where the shape parameter is equal to 1.

Its probability density function (PDF) is:

$f(x; \lambda) = \lambda e^{-\lambda x}$ for $x \geq 0$

where $\lambda > 0$ is the rate parameter.

2.3 Weibull Distribution:

It is a more general distribution with two parameters: shape parameter k and scale parameterλ. Its PDF is:

$$f(x; \lambda, k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-}\left(\frac{x}{\lambda}\right)^{k}, x \geq 0, k > 0$$

2.4 Mean time to Failure:

MTTF is a pivotal reliability metric that estimates the average time a non-repairable asset operates before it fails. This metric is indispensable for asset

management as it allows organizations to predict when a component is likely to fail and plan its replacement proactively.

$$MTTF = \int_0^\infty R(t)dt$$

2.5 Mean time to Repair:

MTTR is a context of human pathology and healthcare refers to the average time required to detect, diagnose, and effectively manage a health issue or medical equipment malfunction to restore a patient to a stable or improved state of health, or to return a medical device to normal operational capacity.

Formula:

$$MTTR = \frac{Total \quad downtime \quad due \quad to \quad human \quad diseases}{Number \quad of \quad treatment \quad actions}$$

2.6 Components of $MTTR$ in Human Pathology:

1.Detection Time: Time taken to identify symptoms and confirm the disease.

2. Diagnosis Time: Time spent determining the pathogen type and severity.

3. Response Time: Time taken to apply treatment.

4. Recovery Time: Time until the Human returns to normal activity or the disease impact is minimized.

III. HUMAN PATHOLOGY SYSTEM

3.1. Human Pathology Problem using Exponential Distribution

We have examined a Human system afflicted by a specific disease in this chapter. A different medication is employed when the first one is unable to treat a certain illness. Assuming X is the ailment, medication 1 is administered first. The second medication is given if the first one is unable to control the illness.

The challenge is to determine the likelihood that a specific Human disease may be prevented using "n" medications. When one medication makes it through the control system, success is achieved. Let us consider a medicine whose time to failure has an exponential distribution with a failure rate parameter λ.

The exponential distribution is a special case of the Weibull distribution, defined by a single rate parameter λ. It is memoryless, meaning the likelihood of a future event is independent of past occurrences.

Probability density function(PDF):

$$f(t) = \lambda e^{-\lambda t}, t \geq 0$$

The i-fold convolution of the probability function.

$g_i(t) = \lambda_i e^{-\lambda_i t}, \quad i = 1,2,3, \cdots, n$

For the second medication at period t, the likelihood function of the system failure time is

If $f_2(t) = \int_0^t g_1(x) g_2(t-x) dx$

where,

$g_1(x)$ represents the time to failure of the first drug up to time x.

$g_2(t-x)$ represents the second medication's time to failure at $(t-x)$.

$$f_2(t) = \int_0^t \lambda e^{-\lambda x} \lambda e^{-\lambda(t-x)} dx$$

$$= \int_0^t \lambda^2 e^{-\lambda x} e^{-\lambda t} e^{\lambda x} dx$$

$$= \lambda^2 t e^{-\lambda t}$$

The 3-fold convolution can be found using

$$f_3(t) = \int_0^t g_2(x) g_3(t-x) dx$$

$$= \int_0^t \lambda^2 x e^{-\lambda x} \lambda e^{-\lambda(t-x)} dx$$

$$f_3(t) = \lambda^3 \frac{t^2}{2} e^{-\lambda t}$$

The 4-fold convolution can be found using

$$f_4(t) = \int_0^t g_3(x) g_4(t-x) dx$$

$$= \int_0^t \lambda^3 \frac{x^2}{2} e^{-\lambda x} \lambda e^{-\lambda(t-x)} dx$$

$$f_4(t) = \frac{\lambda^4}{6} t^3 e^{-\lambda t}$$

In general,

$$f_{n+1}(t) = \frac{\lambda^{n+1}}{n!} t^n e^{-\lambda t}, \quad n = 1,2,3 \cdots$$

3.2 Numerical Examples

Case 1:

When the Human is treated with two medications.

The formula "$R(t) = R_1(t) + R_2(t)$" provides the system's dependability,

where,

$R_1(t) =$ Probability of medicine 1 at time t, 1 is functioning properly.

$R_2(t) =$ Probability of medicine 2 is functioning properly at time t, that is, over the remaining period $(t - x)$, but medication 1 fails before t.

$$R_1(t) = \int_t^\infty g_1(x) dx = \int_t^\infty \lambda e^{-\lambda x} dx$$

$$R_1(t) = e^{-\lambda t}$$

$$R_2(t) = \int_0^t f_1(x) R(t-x) dx$$

$$R_2(t) = \lambda t e^{-\lambda t}$$

Consequently, the system's dependability is provided by

$$R(t) = R_1(t) + R_2(t)$$

$$= e^{-\lambda t} + \lambda t e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}[1 + \lambda t]$$

$$MTTF = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\lambda t}[1 + \lambda t]dt$$

$$MTTF = \frac{2}{\lambda}$$

Table 3.1

t	λ	R(t)	MTTF
2	0.2	0.9384	10
4	0.4	0.5249	5
6	0.6	0.1257	3.33
8	0.8	0.0123	2.5
10	1	0.0005	2

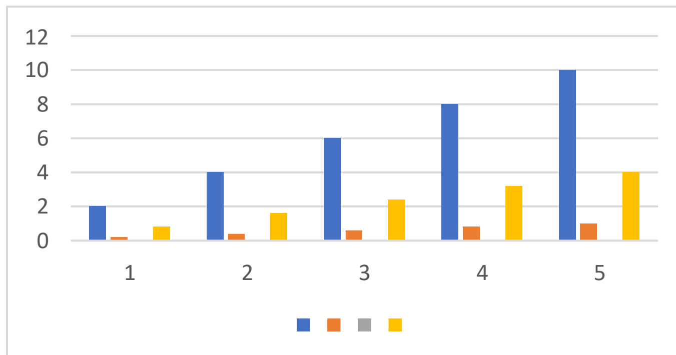

Figure 1: The Case 1 graphical depiction

When the value of t and λ increases, the value of $R(t)$ and the value of $MTTF$ decreases slightly. When the time t and failure rate λ increases, the reliability $R(t)$ and mean time to failure decreases.

Case 2:

When three medications are used on Human System.

$R(t) = R_1(t) + R_2(t) + R_3(t)$

$R_1(t) =$ the probability that medicine 1 is operating at t.

$R_2(t) =$ the probability that medicine 1 failed before medicine drugs 1 and 2 are operating effectively at time t.

$R_3(t) =$ The likelihood that drug 3 is operating correctly at time t and that drugs 1 and 2 fail before t.

$$R_3(t) = \int_0^t f_2(x)R(t - x)dx$$

$$= \int_0^t \lambda^2 x e^{-\lambda x} e^{-\lambda(t-x)} dx$$

$R_3(t) = \frac{\lambda^2}{2} t^2 e^{-\lambda t}$

Thus, the reliability of the system is given by

$R(t) = R_1(t) + R_2(t) + R_3(t)$

$$= e^{-\lambda t} + \lambda t e^{-\lambda t} + \frac{\lambda^2}{2} t^2 e^{-\lambda t}$$

$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2}{2} t^2\right]$$

$$MTTF = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2}{2} t^2\right]dt$$

$$MTTF = \frac{3}{\lambda}$$

Table 3.2

t	λ	R(t)	MTTF
2	0.2	0.9921	15
4	0.4	0.7834	7.5
6	0.6	0.3027	5
8	0.8	0.0463	3.75
10	1	0.0028	3

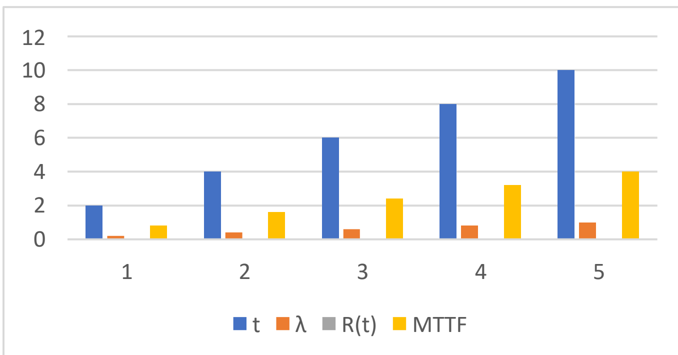

Figure 2: The Case 2 graphical depiction

When the value of t and λ increases slightly, the value of $R(t)$ and the value of $MTTF$ decreases slightly.

When the time t and failure rate λ increases, the reliability of $R(t)$ and mean time to failure decreases.

Case 3:

If the four medications are taken.

$R(t) = R_1(t) + R_2(t) + R_3(t) + R_4(t)$ provides the system's reliability,

where,

$R_1(t) =$ Probability of Medicine at time t, 1 is functioning properly.

$R_2(t) =$ Probability of Medicine 1 fails before t and medication at time t, 2 is functioning properly.

$R_3(t) =$ Probability of Medicine Before t and Medicine 1 and 2 fail at time t, 3 is functioning properly.

$R_4(t) =$ Probability of Medicine Before t and medicine 1, 2 and 3 fail at time t, 4 is operating well.

$$R_4(t) = \int_0^t f_3(x)R(t-x)dx$$

$$= \int_0^t \frac{\lambda^3 x^2}{2} e^{-\lambda x} e^{-\lambda(t-x)} dx$$

$$R_4(t) = \frac{\lambda^3}{6} t^3 e^{-\lambda t}$$

Then,
$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2}{2}t^2 + \frac{\lambda^3}{6}t^3\right]$$

$$MTTF = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2}{2}t^2 + \frac{\lambda^3}{6}t^3\right]dt$$

$$MTTF = \frac{4}{\lambda}$$

Table 3.3

t	λ	R(t)	MTTF
2	0.2	0.9992	20
4	0.4	0.9212	10
6	0.6	0.5152	6.67
8	0.8	0.1189	5
10	1	0.0103	4

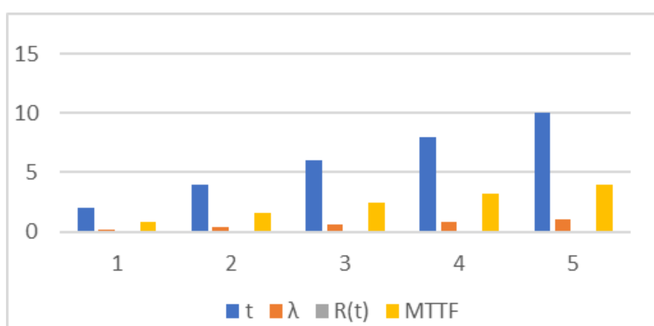

Figure 3: The Case 3 graphical depiction

When the value of t and λ increases slightly, the value of $R(t)$ and the value of $MTTF$ decreases slightly.

When the time t and failure rate λ increases, the reliability of $R(t)$ mean time to failure decreases.

3.3 Human Pathology Problem using Weibull Distribution

The two parameters of the Weibull are the k-shape parameter of λ-scale parameter.

We have examined a plant system afflicted by a specific disease in this chapter. A different medication is employed when the first one is unable to treat a certain illness. Assuming X is the ailment, medication 1 is administered first. The second medication is given if the first one is unable to control the illness.

Determining the probability that a certain Human disease is treated with various approaches is the task at hand. If at least one treatment continues to work in the system, control is attained. Assume that the parameter has a Weibull distribution for the amount of time until a therapy fails.

where k allows the Weibull Distribution of failure rates (increasing, decreasing or constant).

The PDF is

$$f(t;k,\lambda)=\frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1}e^{-\left(\frac{t}{\lambda}\right)^{k}},\quad t>0,\quad k>0$$

If k=1,

$$f(t)=\frac{1}{\lambda}e^{-\frac{t}{\lambda}}, t\geq 0\ ,$$

The system failure time probability function is equivalent to the probability function's i-fold convolution.

The formula is

$$g_i(t)=\frac{1}{\lambda_i}e^{-\frac{t}{\lambda_i}},\quad i=1,2,3,\cdots,n$$

For the second medication at period t, the likelihood function of the system failure time is the equation $\lambda_1=\lambda_2=\lambda_3=\ldots=\lambda_n=\lambda$ [Equal rate of failure].

If $f_2(t)=\int_0^t g_1(x)g_2(t-x)dx$, then

Where,

$g_1(x)$ represents the time to failure of the first drug up to time x as a probability mass function.

$g_2(t-x)$ is the probability mass function of the second medication's time to failure for the time period $(t-x)$.

$$f_2(t) = \int_0^t \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \frac{1}{\lambda} e^{-\frac{(t-x)}{\lambda}} dx$$

$$f_2(t) = t \frac{e^{\frac{-t}{\lambda}}}{\lambda^2}$$

The definition of 3-fold convolution is

$$f_3(t) = \int_0^t g_2(x) g_3(t-x) dx$$

$$= \int_0^t \frac{x}{\lambda^2} e^{-\frac{x}{\lambda}} \frac{1}{\lambda} e^{-\frac{(t-x)}{\lambda}} dx$$

$$= \frac{t^2 e^{\frac{-t}{\lambda}}}{2\lambda^3}$$

The definition of 4-fold convolution is

$$f_4(t) = \int_0^t g_3(x) g_4(t-x) dx$$

$$= \int_0^t \frac{x^2 e^{\frac{-x}{\lambda}}}{2\lambda^3} \frac{1}{\lambda} e^{\frac{-(t-x)}{\lambda}} dx$$

$$f_4(t) = \frac{t^3}{6} \frac{e^{\frac{-t}{\lambda}}}{\lambda^4}$$

In general ,

$$f_{m+1}(t) = \frac{t^m}{m! \lambda^{m+1}} e^{\frac{-t}{\lambda}}, \quad m = 1,2 \cdots$$

3.4 Numerical Examples

Case 1:

When two medications are used.

The formula $R(t) = R_1(t) + R_2(t)$ indicates the system's reliability,

where,

$R_1(t) =$ Medical probability at time t, 1 is functioning properly.

$R_2(t) =$ Probability of medication 1 fails before t and medication 2 is effective at time t.

That is, for the remainder of the time $(t-x)$.

$$R_1(t) = \int_t^\infty g_1(x) dx$$

$$R_1(t) = \int_t^{\infty} \frac{1}{\lambda} e^{\frac{-x}{\lambda}} dx$$

$$R_1(t) = e^{\frac{-t}{\lambda}}$$

$$R_2(t) = \int_0^{t} f_1(x) R(t-x) dx$$

$$R_2(t) = \int_0^{t} \frac{1}{\lambda} e^{\frac{-x}{\lambda}} e^{\frac{-(t-x)}{\lambda}} dx$$

$$R_2(t) = \frac{t e^{\frac{-t}{\lambda}}}{\lambda}$$

Then,

$$R(t) = R_1(t) + R_2(t)$$

$$= e^{\frac{-t}{\lambda}} + \frac{t}{\lambda} e^{\frac{-t}{\lambda}}$$

$$R(t) = e^{\frac{-t}{\lambda}} \left[1 + \frac{t}{\lambda}\right]$$

$$MTTF = \int_0^{\infty} R(t) dt$$

$$= \int_0^{\infty} e^{\frac{-t}{\lambda}} \left[1 + \frac{t}{\lambda}\right] dt$$

$$MTTF = 2\lambda$$

Table 3.4

t	λ	R(t)	MTTF
2	0.2	0.0005	0.4
4	0.4	0.0005	0.8
6	0.6	0.0005	1.2
8	0.8	0.0005	1.6
10	1	0.0005	2

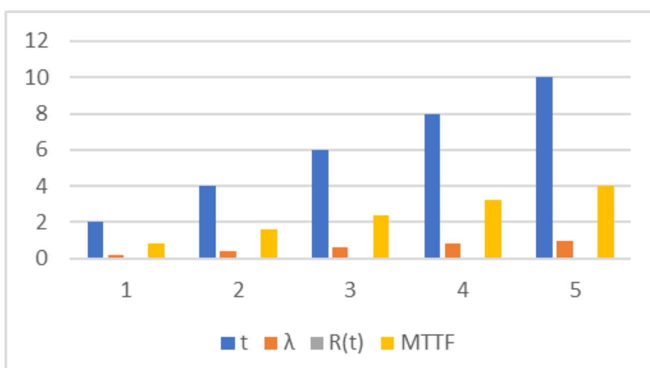

Figure 4: The visual depiction of Case 1

When the value of t and λ increases slightly, the value of $R(t)$ remains constant and the value of $MTTF$ increases slightly.

When the time t and failure rate λ increases, the reliability of $R(t)$ remains constant and mean time to failure increases.

Case 2:

When three medications are used on Human.

The formula $R(t) = R_1(t) + R_2(t) + R_3(t)$ provides the system's reliability,

where,

$R_1(t) =$ Probability of medication at time t, 1 is operating successfully.

$R_2(t) =$ Probability of medication before medication, one failed at time t, medications 1 and 2 are functioning properly.

$R_3(t) =$ the probability that drugs 1 and 2 will stop working before t and that drug 3 will start operating properly at t.

$$R_3(t) = \int_0^t f_2(x)R(t-x)dx$$

$$= \int_0^t \frac{x}{\lambda^2} e^{\frac{-x}{\lambda}} e^{\frac{-(t-x)}{\lambda}} dx$$

$$R_3(t) = \frac{t^2 e^{\frac{-t}{\lambda}}}{2\lambda^2}$$

Then,

$$R(t) = R_1(t) + R_2(t) + R_3(t)$$

$$= e^{\frac{-t}{\lambda}} + \frac{t}{\lambda} e^{\frac{-t}{\lambda}} + \frac{t^2}{2\lambda^2} e^{\frac{-t}{\lambda}}$$

$$R(t) = e^{\frac{-t}{\lambda}} \left[1 + \frac{t}{\lambda} + \frac{t^2}{2\lambda^2}\right]$$

$$MTTF = \int_0^\infty R(t)dt$$

$$= \int_0^{\infty} e^{\frac{-t}{\lambda}} \left[1 + \frac{t}{\lambda} + \frac{t^2}{2\lambda^2}\right] dt$$

$$MTTF = 3\lambda$$

Table:3.5

t	λ	R(t)	MTTF
2	0.2	0.0028	0.6
4	0.4	0.0028	1.2
6	0.6	0.0028	1.8
8	0.8	0.0028	2.4
10	1	0.0028	3

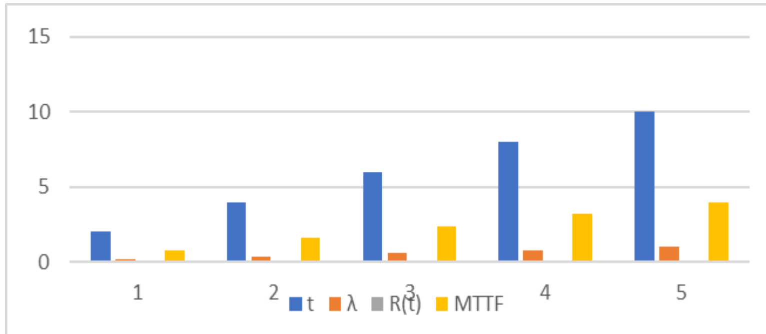

Figure 5: The visual depiction of Case 2

When the value of t and λ increases slightly, the value of $R(t)$ remains constant and the value of $MTTF$ increases slightly.

When the time t and failure rate λ increases, the reliability of $R(t)$ remains constant and mean time to failure increases.

Case 3:

If the four medications are taken,

$R(t) = R_1(t) + R_2(t) + R_3(t) + R_4(t)$ provides the system's reliability,

where ,

$R_1(t) =$ Probability of Medicine at time t, 1 is functioning properly.

$R_2(t) =$ Probability of Medicine 1 fails before t and medication at time t, 2 is functioning properly.

$R_3(t) =$ Probability of Medicine Before t and Medicine 1 and 2 fail at time t, 3 is functioning properly.

$R_4(t) =$ Probability of Medicine Before t and medicine 1, 2 and 3 fail at time t, 4 is operating well.

$$R_4(t) = \int_0^{t} f_3(x) R(t-x) dx$$

$$= \int_0^t \frac{x^2}{2\lambda^2} e^{\frac{-x}{\lambda}} \frac{e^{\frac{-(t-x)}{\lambda}}}{\lambda} dx$$

$$R_4(t) = \frac{t^3 e^{\frac{-t}{\lambda}}}{6\lambda^3}$$

Then,

$R(t) = R_1(t) + R_2(t) + R_3(t) + R_4(t)$

$$= e^{\frac{-t}{\lambda}} + \frac{t}{\lambda} e^{\frac{-t}{\lambda}} + \frac{t^2}{2\lambda^2} e^{\frac{-t}{\lambda}} + \frac{t^3}{6\lambda^3} e^{\frac{-t}{\lambda}}$$

$$R(t) = e^{\frac{-t}{\lambda}} \left[1 + \frac{t}{\lambda} + \frac{t^2}{2\lambda^2} + \frac{t^3}{6\lambda^3}\right]$$

$$MTTF = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{\frac{-t}{\lambda}} \left[1 + \frac{t}{\lambda} + \frac{t^2}{2\lambda^2} + \frac{t^3}{6\lambda^3}\right] dt$$

$$MTTF = 4\lambda$$

Table:3.5:

T	λ	R(t)	MTTF
2	0.2	0.0103	0.8
4	0.4	0.0103	1.6
6	0.6	0.0103	2.4
8	0.8	0.0103	3.2
10	1	0.0103	4

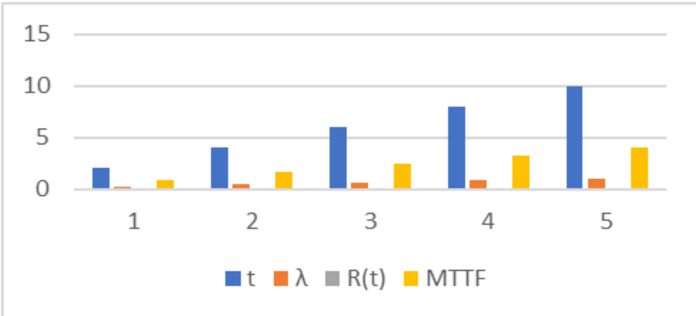

Figure 6: The visual depiction of Case 3

When the value of t and λ increases slightly, the value of $R(t)$ remains constant and the value of $MTTF$ increases slightly.

When the time t and failure rate λ increases, the reliability of $R(t)$ remains constant and mean time to failure increases.

IV. CONCLUSION

This study highlights the critical role of human pathology in supporting reliable healthcare and societal wellbeing. By applying Exponential and Weibull distributions, we analyzed the reliability and MTTF related to human life in a pathology context.Findings show that in the Exponential distribution, increased time and medicine failure rate lead to decreased reliability and MTTF, but Case 3 shows higher values of both compared to Cases 1 and 2. In contrast, in the Weibull distribution, reliability remains constant while MTTF increases across all cases, with Case 3 again showing higher values. Overall, Exponential distribution shows increased human lifetime reliability in all cases, making it more effective in this context.

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