J. Electrical Systems 22-01 (2026): 134-142

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Detection of an Object in a Water Area Based on Vector Hydrophone Data

Abstract: - The article considers the problem of detecting unauthorized objects in a water area using vector hydrophones. A vector hydrophone measures acoustic emissions along three orthogonal components, which makes it possible to determine the direction to a sound source. Based on these data, an algorithm is proposed for calculating the coordinates of the object as a point that minimizes a proximity function to the lines defined by the signal directions. The developed mathematical model takes into account measurement errors and the specific features of sound propagation in water. The proposed approach makes it possible to localize objects in real time using a limited number of sensors and also reveals prospects for improving accuracy when several hydrophones are installed and spectral analysis of signals is applied. Simulation results confirm that the method is suitable for monitoring coastal zones and increasing the security of the water area.

Keywords: Vector hydrophone, Passive sonar, Underwater object detection, Acoustic emission, Sound source localization.

I. INTRODUCTION

One of the important tasks in ensuring the safety and security of facilities in the country's coastal water areas is the protection of the boundaries of the water area. For this purpose, various approaches and technologies are being developed and used. For example, underwater objects can be detected by optical methods: by the reflection or absorption of characteristic green-blue light by the hull of a boat; by the 'traces' of paint particles washed off from the hull; by minute amounts of radioactive substances that are discharged into the water through reactor cooling systems (on nuclear vessels); by the vessel's thermal radiation [1], and so on.

Chinese researchers claim [2] that submarine detection can make use of the wake 'Kelvin wedge' [3] – a V-shaped surface disturbance generated by submarines as they cut through the water. This wake, previously studied for radar-based imaging, produces a weak but detectable magnetic field when the ions of seawater disturbed by the motion of the vessel interact with the Earth's geomagnetic field.

There is a large body of scientific literature devoted to the detection of unauthorized objects using passive and active sonars [4–8].

Among these technologies, the use of vector hydrophones can be noted. A vector hydrophone is an acoustic sensor designed to measure sound radiation in water arriving from all directions or within a specified angular sector [9]. A similar approach is used, for example, in [10, 11]. The output parameters of a vector hydrophone are the intensities (powers) of the incoming acoustic signal along three spatially orthogonal components, which make it possible to determine the vector direction to the object emitting the sound waves.

The aim of this work is to detect objects on the approaches to the water area by their acoustic emissions.

Obviously, in order to jointly process data arriving from several hydrophones, these data must be delivered to a certain water-area monitoring center, and for this it is necessary to use appropriate network equipment that provides their transmission. The data can be transmitted both by cable and wirelessly. The choice of transmission method and network equipment modules must be made taking into account the technical parameters of the hydrophone, operating conditions, distance between the hydrophone and the monitoring center, required reliability, and other factors. On the other hand, since the hydrophones will be located on the bottom of the reservoir, far from infrastructure, their power supply must be ensured, which can be provided either by a separate power cable or via the same network cable used for data exchange [12]. In the article it is assumed that issues related to network equipment and power supply have been resolved, and the present study focuses on developing data processing

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algorithms that make it possible to determine the location of an unauthorized object on the approaches to the water area under consideration.

II. Problem Statement

The problem of detecting unauthorized objects in a water body of known configuration is considered. The accuracy of measuring hydroacoustic signals is crucial for correctly computing the localization of underwater targets, but the complexity of ocean noise often leads to the registration of signals that are heavily contaminated by noise. This noise complicates the accurate estimation of key target parameters such as bearing, range and type. Therefore, effective preliminary noise-suppression processing becomes necessary when handling hydroacoustic signals.

For example, if some known object passes near the hydrophones, its sound can be recorded and taken into account as a background when filtering the acoustic signal from an unauthorized object. The acoustic emissions of active navigation, if it is used within the water area and therefore affects its acoustic background, must also be taken into account.

Since the water area under consideration is sufficiently limited, the curvature of the Earth can be neglected. Therefore we shall assume that the surface of the water area is part of a plane. To specify the locations of objects, we introduce a right-handed rectangular coordinate system Oxyz. For definiteness, we assume that the Oz axis is directed upward, perpendicular to the water surface, and that the Oxy plane coincides with the water surface (Fig. 1).

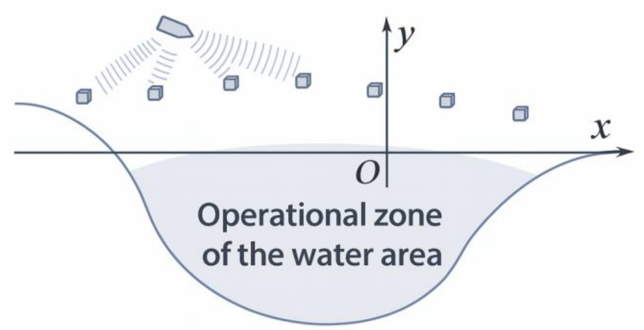

Fig. 1. Schematic location of hydrophones along the boundaries of the operational zone of the water area (top view).

Let N hydrophones be used for monitoring the water area; we number them i = 1, 2, …, N. We assume that all hydrophones are well oriented with respect to the coordinate system Oxyz, that is, their reception vectors for acoustic signals coincide with the directions of the coordinate axes of this system (Fig. 1). The coordinates of the i-th hydrophone with respect to the coordinate system Oxyz will be denoted by (x_i, y_i, z_i).

Suppose that the hydrophones with numbers $i_1, i_2, \dots, i_k$, $2 \le k \le N$, located at points with coordinates $(x_{i_1}, y_{i_1}, z_{i_1})$, $(x_{i_2}, y_{i_2}, z_{i_2})$, … , $(x_{i_k}, y_{i_k}, z_{i_k})$, have registered a noticeable acoustic signal and, after filtering this signal, the computed direction vectors to the newly detected object turned out to be $\mathcal{M}_{i_1} = (m_{x,i_1}, m_{y,i_1}, m_{z,i_1})$, $\mathcal{M}_{i_2} = (m_{x,i_2}, m_{y,i_2}, m_{z,i_2})$, …, $\mathcal{M}_{i_k} = (m_{x,i_k}, m_{y,i_k}, m_{z,i_k})$, respectively. Obviously, if normalization is required, one may assume that the vectors $\mathcal{M}_{i_j}$ have unit length:

$$m_{x,i_j}^2 + m_{y,i_j}^2 + m_{z,i_j}^2 = 1, \qquad j = 1, 2, \dots, k.$$

In the ideal case, all straight lines passing through the points $A_{i_j}\left(x_{i_j}, y_{i_j}, z_{i_j}\right)$ in the directions $\mathcal{M}_{i_j}$ would intersect at a single point where the detected object is located. Let us denote these lines by L_{i_j}. However, there are obviously many factors—among them inaccuracies in determining the hydrophone locations, errors in estimating the direction vectors, refraction of the acoustic signal due to inhomogeneities in the water medium, neglect of the difference in the times of arrival of the acoustic signal at different hydrophones, and so on—because of which these lines may not intersect. Below, we shall consider only those cases in which the vectors $\mathcal{M}_{i_j}$ are pairwise nonparallel.

Let us consider the distance between two skew lines, which can be computed using the formula [13, 14]:

$$D_{i_n,i_j} = \frac{\begin{vmatrix} x_{i_j} - x_{i_n} & y_{i_j} - y_{i_n} & z_{i_j} - z_{i_n} \\ m_{x,i_n} & m_{y,i_n} & m_{z,i_n} \\ m_{x,i_j} & m_{y,i_j} & m_{z,i_j} \end{vmatrix}}{\sqrt{\begin{vmatrix} m_{y,i_n} & m_{z,i_n} \\ m_{y,i_j} & m_{z,i_j} \end{vmatrix}^2 + \begin{vmatrix} m_{z,i_n} & m_{x,i_n} \\ m_{z,i_j} & m_{x,i_j} \end{vmatrix}^2 + \begin{vmatrix} m_{x,i_n} & m_{y,i_n} \\ m_{x,i_j} & m_{y,i_j} \end{vmatrix}^2}}. \quad (1)$$

It is possible that different hydrophones have received signals from different sources. One indication of this is an excessively large spread in the values D_{i_n,i_j}, for example, more than 0.5 nautical miles. If D_{i_n,i_j} does not exceed 0.5 nautical miles, the object can be searched for within a certain region where the lines are closest to each other. Thus, the problem of determining the location of an unauthorized object is reduced to

• choosing a suitable criterion ρ (in the sense of optimization theory, an objective function) for measuring the closeness to all lines $\mathcal{L}_{i_j}$, respectively passing through the points A_{i_j} in the direction of the vector $\mathcal{M}_{i_j}$, where only those hydrophones for which $D_{i_n,i_j} < 0.5$ nautical miles are taken into account;

• describing an algorithm for finding the point that is closest to all lines $\mathcal{L}_{i_j}$ in the sense of the ρ measure (that is, that minimizes the objective function).

III. Mathematical Model

In order to define an objective function that provides an estimate of the closeness between two points $A_1(x_1, y_1, z_1)$ and $A_2(x_2, y_2, z_2)$, it is necessary to introduce a certain positive function $\rho(A_1, A_2)$ [15].

For example, if we denote by $d(A_1, A_2)$ the classical Euclidean metric [16, p. 21], i.e.

$$d(A_1, A_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}, \quad (2)$$

then as the function $\rho(A_1, A_2)$ one can take, for instance,

$$\rho(A_1, A_2) \equiv d^{\theta}(A_1, A_2), \ \ \theta \geq 1. \quad (3)$$

Then the measure of closeness between a point A_1 and a line $\mathcal{L}$ can be defined by

$$\rho(A_1, \mathcal{L}) \equiv \min_{A_2 \in \mathcal{L}} \{\rho(A_1, A_2)\}. \quad (4)$$

Let $\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_k$ be some, in general (as was shown in the previous section for the lines), non-intersecting lines in three-dimensional space $\mathcal{R}^3$, and let $A \in \mathcal{R}^3$. In accordance with definition (4), we denote the measure of closeness from the point A to the line $\mathcal{L}_j$, $(j = 1, 2, \dots, k)$ by $\rho_j(A) \equiv \rho(A, \mathcal{L}_j)$.

We shall regard as the presumed (or approximate) position of the object such a point $A \in \mathcal{R}^3$ for which the objective function

$$J(A) \equiv \sum_{j=1}^{n} \rho_j(A)$$

attains the smallest value. It should be noted that the power of a sound signal in sea water decays rapidly (exponentially) with distance; therefore, to determine the source of the sound it is sufficient to use no more than three hydrophones.

Let $A_1(x_1, y_1, z_1)$ and $A_2(x_2, y_2, z_2)$ be the two closest points on two skew lines representing the directions of the signal from the unauthorized object to the hydrophones. A natural question arises: can we, on the basis of this information, infer where the unauthorized object is most likely located? To answer this question, it is necessary to introduce a suitable metric for estimating the distance between points.

The purpose of this work is not to test all metrics described in mathematical analysis for suitability, so we shall restrict ourselves to the simplest metrics from the computational point of view. In addition, in order to be able to apply the tools of differential calculus when searching for the minimum of these functions, we give preference, when choosing a metric, to smooth functions. In this sense, as a measure of closeness we shall further consider the function ρ described in (3), generated by the usual Euclidean metric for $\theta = 1$ or $\theta = 2$.

Comparison of proximity measures for θ. For simplicity of understanding, we examine, for $\theta = 1, 2,$ the specific problem of finding a point A with coordinates (x, y, z) such that the total proximity measure to two known points A_1 and A_2 is minimal. Let us first consider the case $\theta = 1$. To find the minimum value of the objective function

$$J(A) = \sum_{j=1}^{2} \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + \left(z - z_j\right)^2},$$

we apply the least squares method.

To do this, we first compute the partial derivatives $\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y}, \frac{\partial J}{\partial z}$ and set them equal to zero:

$$\begin{cases} \displaystyle\sum_{j=1}^{2} \frac{x - x_j}{\sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + \left(z - z_j\right)^2}} = 0\,, \\ \displaystyle\sum_{j=1}^{2} \frac{y - y_j}{\sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + \left(z - z_j\right)^2}} = 0\,, \\ \displaystyle\sum_{j=1}^{2} \frac{z - z_j}{\sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + \left(z - z_j\right)^2}} = 0\,. \end{cases} \tag{5}$$

Rewriting this system in the form

$$\begin{cases} (x - x_1)\sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2} + (x - x_2)\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 0, \\ (y - y_1)\sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2} + (y - y_2)\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 0, \\ (z - z_1)\sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2} + (z - z_2)\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 0 \end{cases}$$

we obtain the following equation, which represents the equation of the line passing through the points A_1 and A_2:

$$\frac{x - x_1}{x - x_2} = \frac{y - y_1}{y - y_2} = \frac{z - z_1}{z - z_2}. \tag{6}$$

It is easy to see that, for any real parameter $\lambda \in [0, 1]$, the point

$$(x_\lambda, y_\lambda, z_\lambda) \equiv (\lambda x_1 + (1 - \lambda)x_2, \lambda y_1 + (1 - \lambda)y_2, \lambda z_1 + (1 - \lambda)z_2)$$

belongs to the line (6) and satisfies system (5). Consequently, for $\theta = 1$ this objective function does not yield a unique point of likely localization of the object.

Now let $\theta = 1$. As the objective function we choose

$$J(A) = \sum_{j=1}^{2} \left[\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + \left(z - z_j\right)^2\right].$$

We again apply the least squares method:

$$\frac{\partial J}{\partial x} = 2\sum_{j=1}^{2}\left(x - x_j\right) = 2\left[2x - \sum_{j=1}^{2} x_j\right] = 0,$$

$$\frac{\partial J}{\partial y} = 2\sum_{j=1}^{2}\left(y - y_j\right) = 2\left[2y - \sum_{j=1}^{2} y_j\right] = 0,$$

$$\frac{\partial J}{\partial z} = 2\sum_{j=1}^{2}(z - z_j) = 2\left[2z - \sum_{j=1}^{2} z_j\right] = 0.$$

From these equations, the estimated coordinates of the object are uniquely determined by the formulas

$$\begin{cases} x = \frac{1}{2}\sum_{j=1}^{2} x_j\,, \\ y = \frac{1}{2}\sum_{j=1}^{2} y_j\,, \\ z = \frac{1}{2}\sum_{j=1}^{2} z_j\,. \end{cases}$$

Thus, as a proximity measure satisfying the requirements formulated above, one can choose the function (3) for $\theta = 2$.

Algorithm for finding the coordinates of the most probable location point. As already noted above, when a sound signal propagates in water its power decreases exponentially with increasing distance from the source. Therefore, when calculating the coordinates of the sound source it is reasonable to restrict ourselves to three hydrophones that receive the acoustic signal with relatively stable power. To determine the coordinates of the probable position of the object, we use the proximity function (3) with $\theta = 2$ for the hydrophone observations $\mathcal{M}_j = (m_{x,j}, m_{y,j}, m_{z,j}), j = 1, 2, 3$.

Introducing a parameter $t_j \in \mathcal{R}^1, j = 1, 2, 3$, we write the t-parametric equations of the straight lines that pass through the hydrophone positions $A_j(x_j, y_j, z_j)$ in the direction of the corresponding hydrophone observation vector $\mathcal{M}_j$ [16, p. 136] in the form

$$\begin{cases} x = m_{x,j}t_j + x_j, \\ y = m_{y,j}t_j + y_j,. \\ z = m_{z,j}t_j + z_j. \end{cases} \qquad (7)$$

Then the coordinates of the probable location of the object can be determined by minimizing the following objective function:

$$J(x, y, z, t_1, t_2, t_3) \equiv \sum_{j=1}^{3}\left[(x - m_{x,j}t_j - x_j)^2 + (y - m_{y,j}t_j - y_j)^2 + (z - m_{z,j}t_j - z_j)^2\right].$$

Applying the method of least squares, we compute the partial derivatives $\frac{\partial J}{\partial x}, \frac{\partial J}{\partial y}, \frac{\partial J}{\partial z}, \frac{\partial J}{\partial t_j}$ and set them equal to zero:

$$\sum_{j=1}^{3}(m_{x,j}t_j + x_j - x) = 0,$$

$$\sum_{j=1}^{3}(m_{y,j}t_j + y_j - y) = 0,$$

$$\sum_{j=1}^{3}(m_{z,j}t_j + z_j - z) = 0,$$

$$m_{x,j}(m_{x,j}t_j + x_j - x) + m_{y,j}(m_{y,j}t_j + y_j - y) + m_{z,j}(m_{z,j}t_j + z_j - z) = 0, \qquad j = 1, 2, 3.$$

Given that $m_{x,j}^2 + m_{y,j}^2 + m_{z,j}^2 = 1$, we obtain:

$$\sum_{j=1}^{3} m_{x,j} t_j + \sum_{j=1}^{3} x_j - 3x = 0,$$

$$\sum_{j=1}^{3} m_{y,j} t_j + \sum_{j=1}^{3} y_j - 3y = 0,$$

$$\sum_{j=1}^{3} m_{z,j} t_j + \sum_{j=1}^{3} z_j - 3z = 0,$$

$$t_j = m_{x,j} x + m_{y,j} y + m_{z,j} z - \left(m_{x,j} x_j + m_{y,j} y_j + m_{z,j} z_j\right).$$

Substituting the expression t_j into the previous equations, we obtain the following system of linear algebraic equations in x, y, z:

$$\begin{cases} x\left(\sum_{j=1}^{3} m_{x,j}^2 - 3\right) + y\sum_{j=1}^{3} m_{y,j} m_{x,j} + z\sum_{j=1}^{3} m_{z,j} m_{x,j} = \sum_{j=1}^{3}\left(m_{x,j}^2 x_j + m_{y,j} m_{x,j} y_j + m_{z,j} m_{x,j} z_j - x_j\right), \\ x\sum_{j=1}^{3} m_{x,j} m_{y,j} + y\left(\sum_{j=1}^{3} m_{y,j}^2 - 3\right) + z\sum_{j=1}^{3} m_{z,j} m_{y,j} = \sum_{j=1}^{3}\left(m_{x,j} m_{y,j} x_j + m_{y,j}^2 y_j + m_{z,j} m_{y,j} z_j - y_j\right), \\ x\sum_{j=1}^{3} m_{x,j} m_{z,j} + y\sum_{j=1}^{3} m_{y,j} m_{z,j} + z\left(\sum_{j=1}^{3} m_{z,j}^2 - 3\right) = \sum_{j=1}^{3}\left(m_{x,j} m_{z,j} x_j + m_{y,j} m_{z,j} y_j + m_{z,j}^2 z_j - z_j\right). \end{cases} \quad (8)$$

The main determinant of this system is

$$\Delta \equiv \begin{vmatrix} \sum_{j=1}^{3} m_{x,j}^2 - 3 & \sum_{j=1}^{3} m_{y,j} m_{x,j} & \sum_{j=1}^{3} m_{z,j} m_{x,j} \\ \sum_{j=1}^{3} m_{x,j} m_{y,j} & \sum_{j=1}^{3} m_{y,j}^2 - 3 & \sum_{j=1}^{3} m_{z,j} m_{y,j} \\ \sum_{j=1}^{3} m_{x,j} m_{z,j} & \sum_{j=1}^{3} m_{y,j} m_{z,j} & \sum_{j=1}^{3} m_{z,j}^2 - 3 \end{vmatrix} = S^2(\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3) - $$
$$-\frac{3}{2}\sum_{\substack{j_1, j_2 = 1 \\ j_1 \neq j_2}}^{3}\left(\begin{vmatrix} m_{x,j_1} & m_{x,j_2} \\ m_{y,j_1} & m_{y,j_2} \end{vmatrix}^2 + \begin{vmatrix} m_{x,j_1} & m_{x,j_2} \\ m_{z,j_1} & m_{z,j_2} \end{vmatrix}^2 + \begin{vmatrix} m_{y,j_1} & m_{y,j_2} \\ m_{z,j_1} & m_{z,j_2} \end{vmatrix}^2\right), \quad (9)$$

where

$$S(\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3) = \begin{vmatrix} m_{x,1} & m_{x,2} & m_{x,3} \\ m_{y,1} & m_{y,2} & m_{y,3} \\ m_{z,1} & m_{z,2} & m_{z,3} \end{vmatrix}$$

is the mixed, or triple, scalar product of the vectors $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$, which is equal to the volume of the three-dimensional parallelepiped spanned by these vectors, taken with a plus sign if the vectors $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$ form a right-handed triple and with a minus sign if they form a left-handed triple [16, p. 73]. Due to errors in the hydrophone data, the determinant (9) may vanish. However, if $|\Delta| \neq 0$, then system (8) has a unique solution by Cramer's rule:

$$x = \frac{\Delta_x}{\Delta}, \qquad y = \frac{\Delta_y}{\Delta}, \qquad z = \frac{\Delta_z}{\Delta}. \quad (10)$$

Formulas for calculating Δ_x, Δ_y and $\Delta_z$are given below:

$$\Delta_x = \begin{vmatrix} \sum_{j=1}^{3}\left(m_{x,j}^2 x_j + m_{y,j}m_{x,j}y_j + m_{z,j}m_{x,j}z_j - x_j\right) & \sum_{j=1}^{3} m_{y,j}m_{x,j} & \sum_{j=1}^{3} m_{z,j}m_{x,j} \\ \sum_{j=1}^{3}\left(m_{x,j}m_{y,j}x_j + m_{y,j}^2 y_j + m_{z,j}m_{y,j}z_j - y_j\right) & \sum_{j=1}^{3} m_{y,j}^2 - 3 & \sum_{j=1}^{3} m_{z,j}m_{y,j} \\ \sum_{j=1}^{3}\left(m_{x,j}m_{z,j}x_j + m_{y,j}m_{z,j}y_j + m_{z,j}^2 z_j - z_j\right) & \sum_{j=1}^{3} m_{y,j}m_{z,j} & \sum_{j=1}^{3} m_{z,j}^2 - 3 \end{vmatrix} =$$

$$= \sum_{\substack{j_1,j_2,j_3=1 \\ j_1\neq j_2\neq j_3\neq j_1}}^{3} m_{y,j_2}m_{z,j_3}\left(m_{x,j_1}x_{j_1} + m_{y,j_1}y_{j_1} + m_{z,j_1}z_{j_1}\right)S\left(\mathcal{M}_{j_1}, \mathcal{M}_{j_2}, \mathcal{M}_{j_3}\right) - 9\sum_{j=1}^{3} x_j,$$

$$\Delta_y = \begin{vmatrix} \sum_{j=1}^{3} m_{x,j}^2 - 3 & \sum_{j=1}^{3}\left(m_{x,j}^2 x_j + m_{y,j}m_{x,j}y_j + m_{z,j}m_{x,j}z_j - x_j\right) & \sum_{j=1}^{3} m_{z,j}m_{x,j} \\ \sum_{j=1}^{3} m_{x,j}m_{y,j} & \sum_{j=1}^{3}\left(m_{x,j}m_{y,j}x_j + m_{y,j}^2 y_j + m_{z,j}m_{y,j}z_j - y_j\right) & \sum_{j=1}^{3} m_{z,j}m_{y,j} \\ \sum_{j=1}^{3} m_{x,j}m_{z,j} & \sum_{j=1}^{3}\left(m_{x,j}m_{z,j}x_j + m_{y,j}m_{z,j}y_j + m_{z,j}^2 z_j - z_j\right) & \sum_{j=1}^{3} m_{z,j}^2 - 3 \end{vmatrix} =$$

$$= \sum_{\substack{j_1,j_2,j_3=1 \\ j_1\neq j_2\neq j_3\neq j_1}}^{3} m_{x,j_1}m_{z,j_3}\left(m_{x,j_2}x_{j_2} + m_{y,j_2}y_{j_2} + m_{z,j_2}z_{j_2}\right)S\left(\mathcal{M}_{j_1}, \mathcal{M}_{j_2}, \mathcal{M}_{j_3}\right) - 9\sum_{j=1}^{3} y_j,$$

$$\Delta_z = \begin{vmatrix} \sum_{j=1}^{3} m_{x,j}^2 - 3 & \sum_{j=1}^{3} m_{y,j}m_{x,j} & \sum_{j=1}^{3}\left(m_{x,j}^2 x_j + m_{y,j}m_{x,j}y_j + m_{z,j}m_{x,j}z_j - x_j\right) \\ \sum_{j=1}^{3} m_{x,j}m_{y,j} & \sum_{j=1}^{3} m_{y,j}^2 - 3 & \sum_{j=1}^{3}\left(m_{x,j}m_{y,j}x_j + m_{y,j}^2 y_j + m_{z,j}m_{y,j}z_j - y_j\right) \\ \sum_{j=1}^{3} m_{x,j}m_{z,j} & \sum_{j=1}^{3} m_{y,j}m_{z,j} & \sum_{j=1}^{3}\left(m_{x,j}m_{z,j}x_j + m_{y,j}m_{z,j}y_j + m_{z,j}^2 z_j - z_j\right) \end{vmatrix} =$$

$$= \sum_{\substack{j_1,j_2,j_3=1 \\ j_1\neq j_2\neq j_3\neq j_1}}^{3} m_{x,j_1}m_{y,j_2}\left(m_{x,j_3}x_{j_3} + m_{y,j_3}y_{j_3} + m_{z,j_3}z_{j_3}\right)S\left(\mathcal{M}_{j_1}, \mathcal{M}_{j_2}, \mathcal{M}_{j_3}\right) - 9\sum_{j=1}^{3} z_j.$$

Assuming that the components of the vectors $\mathcal{M}_j$ $(j = 1, 2, 3)$ are determined with some relative error δ_m, and the accuracy of calculating the hydrophone coordinates is δ_s, then the error in calculating the coordinates of the sound source for $|\Delta| > \Delta_0$ will be on the order of $(2\delta_m + \delta_s)/\Delta_0$.

IV. Location of a Surface Object

In the case when, by the nature of the sound signal, the object is recognized as a surface vessel, the coordinates of its likely position are computed more easily.

Since in this case the object floats on the surface of the body of water, its z-coordinate is zero. Consequently, from equations (7) we find $t_j = -z_j/m_{z,j}$ and

$$\begin{cases} x = x_j - \dfrac{m_{x,j}}{m_{z,j}} z_j, \\ y = y_j - \dfrac{m_{y,j}}{m_{z,j}} z_j. \end{cases}$$

Hence, the likely point of the object's location can be taken as the point with coordinates

$$\begin{cases} x = \frac{1}{n}\sum_{j=1}^{n}\left(x_j - \frac{m_{x,j}}{m_{z,j}} z_j\right), \\ y = \frac{1}{n}\sum_{j=1}^{n}\left(y_j - \frac{m_{y,j}}{m_{z,j}} z_j\right). \end{cases}$$

V. Example of Computing the Coordinates of an Unauthorized Object

Let the vector hydrophones be located at points with coordinates $A_1(-1.00, -0.01, -0.20)$, $A_2(+0.00, +0.00, -0.30)$ and $A_3(1.00, +0.00, -0.25)$, expressed in nautical miles. Assuming that the unauthorized object is located at point $A_0(0.15, 0.25, -0.12)$, the vectors specifying the direction to the object were calculated:

$$\begin{cases} \mathcal{M}_1^0 = (+0.9731446, +0.2200153, +0.0676970), \\ \mathcal{M}_2^0 = (+0.4377813, +0.7296355, +0.5253376), \\ \mathcal{M}_3^0 = (-0.9492024, +0.2791772, +0.1451721). \end{cases}$$

Using these values as a basis, we generated the following "hydrophone data" with some random deviations within 1%.

$$\begin{cases} \mathcal{M}_1 = (+0.97, +0.21, +0.07), \\ \mathcal{M}_2 = (+0.45, +0.73, +0.52), \\ \mathcal{M}_3 = (-0.94, +0.27, +0.15). \end{cases}$$

Applying formula (10), we find the estimated location of the unauthorized object:

$$A(+0.15379, +0.24357, -0.11840).$$

It is obvious that the distance, computed using formula (2), between the true position of the unauthorized object A_0 and the estimated position

$$d(A, A_0) = 0.0076$$

is on the order of a few hundredths of a nautical mile, which can be regarded as an acceptable computational error. Therefore, the calculated coordinates of point A may be considered sufficiently accurate.

VI. Conclusion

In the article, a method for detecting unauthorized objects in a water area based on vector hydrophone data has been developed and investigated. The proposed mathematical model and localization algorithm make it possible to compute the coordinates of the acoustic emission source with sufficient accuracy even in the presence of acoustic noise and measurement errors. It has been shown that the use of vector hydrophones offers a number of advantages over traditional scalar hydrophones: the computation process is simplified, there is no need for measurement synchronization, and the reliability and accuracy of monitoring are improved.

The simulation results have confirmed the operability of the approach and the possibility of its practical implementation for the control of coastal zones. In the future, the use of several vector hydrophones and spectral analysis of sound signals will make it possible not only to refine the locations of objects, but also to carry out their preliminary identification. Thus, the proposed system can become an effective tool for ensuring security in a water area.

Acknowledgment

This publication was carried out under project G7475, funded by the NATO Science for Peace and Security Programme.

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