The aims of this paper is to present an accurate two-axis model for the analysis and diagnosis of dual-stator induction machine under healthy and windings electrical faults conditions. While incorporating the stator and rotor windings functions, the developed model is based on the magnetically coupled electric circuits equations expressed in two different coordinate systems, and combined with the mechanical equations through the electromagnetic torque. The obtained time-differential state equations system is implemented under Matlab environment and numerically solved using the fourth order Rung-Kutta method with variable step time corrected at each rotor displacement. The computer simulation model can be used under unbalanced parameters or asymmetries operating conditions due to faults. A dual stator induction machine is simulated under healthy operating, and faults due to stator electrical inter-turns short-circuits, or stator and rotor broken phases conditions. The studied faults impact is analyzed while focusing on the currents, electromagnetic torque and rotor speed behaviors.

Keywords: Dual stator induction machine, electric circuits magnetically coupled model, inter-turn short circuits, stator phases breakdown, broken rotor bars.

1. Nomenclature

$s_1$, $s_2$ are respectively the indexes of the first star and second star of the stator windings.

$r$ is related to the rotor index.

$L^s_{a,b,c}, l^s_f$ are respectively the self and leakage inductances of the stator windings

$L_r, l^r_f$ are respectively the self and leakage inductances of the rotor windings

$R_s, R_r$ are respectively the resistance of the stator/rotor

$m_{sr}$ is the mutual inductance between stator and rotor windings.

$m_{s_1s_2}$ is the mutual inductance between stator windings.

$\omega_r, \theta_r$ are the rotor speed and the angular position.

$C_{em}, C_{load}$ are respectively the electromagnetic and load torque.

$J, f$ are respectively the shaft inertia and the friction coefficient

$u^s_{cc}, i^s_{cc}$ are respectively the voltage and current of short-circuits winding.

$u^s_o, i^s_o$ are respectively the voltage and current of broken winding.

2. Introduction

Multiphases electrical machines are finding favor in some variable-speed drive and specialized critical applications where high reliability and security is demanded such as electric/hybrid vehicles, aerospace applications, ship propulsion, and high power applications. They possess several advantageous over conventional three-phases drives.
counterparts such as: reducing the amplitude and increasing the frequency of pulsating torques, reducing the rotor harmonic currents and the stator per phase currents without increasing the voltages per phases and the power in same frame [1-2].

Among of the multiphases machines, the most used are the Dual-Stator Induction Machines (DSIM) offering power segmentation are very attractive for high power applications, since they allow the use of lower rating power electronic devices able to function at a high switching frequency. Nonetheless, failures can appear, so we have to monitor the process or the multiphase machines and there (DSIM) type in order to limit the dangerousness of the defective process and to limit the coast of the maintenance drives in many branches of the industry. As a consequence of electrical or mechanical faults, which may arise during the operation period, many electrical machines are operating at asymmetrical conditions. The basis of reliable diagnosis is an understanding of the electric, magnetic and mechanical behavior of the machine in healthy and under fault conditions [3-4].

In general, condition monitoring schemes have concentrated on sensing specific failures modes in one of three induction motors components: the stator, the rotor, or the bearings. Even though thermal and vibration monitoring have been utilized for decades, most of the recent research has been directed toward electrical monitoring of the motor with emphasis on inspecting the stator current of the motor. All of the presently available techniques require the user to have some degree of expertise in order to distinguish a normal operating condition from a potential failure mode [4-6].

The aims of the present paper, is to develop an accurate transient model of Dual-Star Induction Machines (DSIM) under healthy and windings faults conditions operations. The proposed original phase axes model is based on a coupled magnetic circuits approach. The original phases axes model equations is based on the expression of elementary electric equations of each stator and rotor windings, respectively in the fixed coordinate system, and the rotating coordinate one. Both equations are associated to the mechanical equations through the torque, and then a differential equations system model is obtained.

The model establishes a relation between a state vector formed by the windings currents, the speed, the angular displacement, and the external rated voltages through the mechanical parameters, the constant resistances and self-inductances matrices, and particularly the mutual inductance-angular displacement dependence matrices. The model can be used when the windings parameters of the phases are not balanced with very few restrictive assumptions. The cases of (DSIM) under healthy, stator windings inter-turns short circuits, stator phase breakdown, and broken bars are studied.

3. Healthy Model of Dual-Stator-IM

Consider a (DSIM) in which the two stator stars windings connection are spatially displaced by a fixed electrical angle $\alpha$, classically shifted to be equal to 30°. A (DSIM) has two three-phase stator windings sets, which will be referred as “stators” throughout this paper for conciseness. The windings are modelled as a serials resistance and electromotives forces, such as showed by the “Fig.1”. The assumptions made concern the infinite permeability of iron, uniform air gap, stator windings identical, negligible slotting effect [7].

In general the servo-motor is controlled in the synchronously rotating reference frame to obtain the couplings between the motor current and the torque. In the case of the control without position sensor, the servo-motor do not possess any information on the position $\theta$. 

417x64]
In that case the α–β axis are supposed to replace the d-q axis in the control. Supposing that the axis α is necessarily non adjusted with the axis d, the rotor angle θ is defined as the angle between the axis α and the phase U of the motor.

The control is achieved by the estimation of the rotor position θ.

![Fig. 1: Windings configuration of the Dual-Stator Induction Machine.](image)

### 3.1 Native Mathematical Model

According to basic circuit theory of the Kirchhoff’s laws and the classical assumptions, the (DSIM) original phase axes model can be expressed as follows:

\[
\begin{bmatrix}
\dot{u}^s \\
\dot{u}^r
\end{bmatrix} =
\begin{bmatrix}
R_{ss} & [0]_{6 \times 3} \\
[0]_{3 \times 6} & R_{rr}
\end{bmatrix}
\begin{bmatrix}
I^s \\
I^r
\end{bmatrix}
+ \frac{d}{dt}
\begin{bmatrix}
\phi^s \\
\phi^r
\end{bmatrix} =
\begin{bmatrix}
\dot{\phi}^s \\
\dot{\phi}^r
\end{bmatrix} =
\begin{bmatrix}
\dot{\phi}_{SRM}
\end{bmatrix}
\tag{1}
\]

\[
\begin{bmatrix}
\phi^s \\
\phi^r
\end{bmatrix} =
\begin{bmatrix}
L_{ss} & [M_{\text{SR}}(\theta)]_{3 \times 6} \\
[M_{\text{SR}}(\theta)]_{3 \times 6} & L_{rr}
\end{bmatrix}
\begin{bmatrix}
I^s \\
I^r
\end{bmatrix} =
\begin{bmatrix}
\dot{I}^s \\
\dot{I}^r
\end{bmatrix} =
\begin{bmatrix}
\dot{I}_{\text{SR}}
\end{bmatrix}
\tag{2}
\]

Where the elementary vector voltage and current are defined, such as:

\[
\begin{bmatrix}
\dot{u}^s \\
\dot{u}^r
\end{bmatrix} =
\begin{bmatrix}
\dot{u}_{a_1}^s, \dot{u}_{b_1}^s, \dot{u}_{c_1}^s, \dot{u}_{a_2}^s, \dot{u}_{b_2}^s, \dot{u}_{c_2}^s
\end{bmatrix} \\
\begin{bmatrix}
\dot{u}_{A}^r, \dot{u}_{B}^r, \dot{u}_{C}^r
\end{bmatrix}
\tag{3}
\]

\[
\begin{bmatrix}
\dot{I}^s \\
\dot{I}^r
\end{bmatrix} =
\begin{bmatrix}
\dot{I}_{a_1}^s, \dot{I}_{b_1}^s, \dot{I}_{c_1}^s, \dot{I}_{a_2}^s, \dot{I}_{b_2}^s, \dot{I}_{c_2}^s
\end{bmatrix} \\
\begin{bmatrix}
\dot{I}_{A}^r, \dot{I}_{B}^r, \dot{I}_{C}^r
\end{bmatrix}
\tag{4}
\]

The matrices resistance, self-inductance of the stator and the rotor windings, and their mutual inductance matrices are respectively: \([R_{ss}],[L_{ss}],[R_{rr}],[L_{rr}]\) and \([M_{\text{SR}}(\theta)]\).
Since the windings resistance values for the stator and the rotor are not necessarily equal, the stator and rotors resistances matrices are given such as:

\[
\begin{bmatrix}
R_{a_1}^s & 0 & 0 & 0 & 0 & 0 \\
0 & R_{b_1}^s & 0 & 0 & 0 & 0 \\
0 & 0 & R_{c_1}^s & 0 & 0 & 0 \\
0 & 0 & 0 & R_{d_1}^s & 0 & 0 \\
0 & 0 & 0 & 0 & R_{e_1}^s & 0 \\
0 & 0 & 0 & 0 & 0 & R_{f_1}^s
\end{bmatrix}
\]

\[
[R_{rr}]_{3 \times 3} = \begin{bmatrix} R_{r_1} & 0 & 0 \\ 0 & R_{r_2} & 0 \\ 0 & 0 & R_{r_3} \end{bmatrix} \quad (3.a)
\]

Let us defined the angle \( \beta = 2\pi/3 \) between the two rotor windings, and the two stator windings of each stars 1 or 2. The stator and rotor self and mutual inductance matrices are written as:

\[
[L_{ss}]_{6 \times 6} = m_{sl2} \begin{bmatrix}
1 & \cos(\beta) & \cos(-\beta) & \cos(\alpha) & \cos(\alpha + \beta) & \cos(\alpha - \beta) \\
\cos(-\beta) & 1 & \cos(\beta) & \cos(\alpha - \beta) & \cos(\alpha) & \cos(\alpha + \beta) \\
\cos(\beta) & \cos(-\beta) & 1 & \cos(\alpha + \beta) & \cos(\alpha - \beta) & \cos(\alpha) \\
\cos(\alpha) & \cos(\alpha - \beta) & \cos(\alpha + \beta) & 1 & \cos(\beta) & \cos(-\beta) \\
\cos(\alpha + \beta) & \cos(\alpha) & \cos(\alpha - \beta) & \cos(-\beta) & 1 & \cos(\beta) \\
\cos(\alpha - \beta) & \cos(\alpha + \beta) & \cos(\alpha) & \cos(\beta) & \cos(-\beta) & 1
\end{bmatrix}
\]

\[
[L_{a_1}^s] \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & L_{b_1}^s & 0 & 0 & 0 \\
0 & 0 & L_{c_1}^s & 0 & 0 \\
0 & 0 & 0 & L_{d_1}^s & 0 \\
0 & 0 & 0 & 0 & L_{e_1}^s \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} +
\]

\[
[M_{SR}(\theta_r)]_{3 \times 6} = m_{sr} \begin{bmatrix}
\cos(\theta_r) & \cos(\theta_r - \beta) & \cos(\theta_r + \beta) \\
\cos(\theta_r + \beta) & \cos(\theta_r) & \cos(\theta_r - \beta) \\
\cos(\theta_r - \beta) & \cos(\theta_r + \beta) & \cos(\theta_r) \\
\cos(\theta_r - \alpha) & \cos(\theta_r - (\alpha + \beta)) & \cos(\theta_r - (\alpha - \beta)) \\
\cos(\theta_r - (\alpha + \beta)) & \cos(\theta_r - \alpha) & \cos(\theta_r - (\alpha + \beta)) \\
\cos(\theta_r - (\alpha - \beta)) & \cos(\theta_r - (\alpha + \beta)) & \cos(\theta_r - \alpha)
\end{bmatrix} \quad (3.c)
\]

\[
[L_{rr}]_{3 \times 3} = m_{rr} \begin{bmatrix}
1 & \cos(\beta) & \cos(-\beta) \\
\cos(-\beta) & 1 & \cos(\beta) \\
\cos(\beta) & \cos(-\beta) & 1
\end{bmatrix} \begin{bmatrix} L_{r_1} & 0 & 0 \\ 0 & L_{r_2} & 0 \\ 0 & 0 & L_{r_3} \end{bmatrix} \quad (3.d)
\]

To the set of (1) and (2) equations, one must add the mechanical equations of the shaft:

\[
\frac{d\omega_r}{dt} = \frac{1}{J} \left( C_{em} - C_{load} - f\omega_r \right) \quad (4)
\]
\[
\frac{d\theta_r}{dt} = \omega_r 
\]  \hspace{1cm} (5)

With \(\omega_r, \theta_r\) denote the rotor speed and the angular position, \(C_{em}, C_{load}\) are respectively the electromagnetic and load torque, and \(J, f\) are respectively the shaft inertia and the friction coefficient.

The electromagnetic torque \(C_{em}\) is obtained from the magnetic stored co-energy \(W_{co}\).

\[
W_{co} = \frac{1}{2} \left[ I_s \right]^T \left[ L_{ss} \right] \left[ L_{sr} \right]^T \left[ M_{SR} (\theta_r) \right] \left[ I_s \right] + \frac{1}{2} \left[ I_r \right]^T \left[ L_{rr} \right] \left[ I_r \right] + \frac{1}{2} \left[ I_s \right]^T \left[ M_{SR} (\theta_r) \right] \left[ I_r \right] + \frac{1}{2} \left[ I_r \right]^T \left[ M_{SR} (\theta_r) \right] \left[ I_s \right] 
\]  \hspace{1cm} (6)

It is obvious that the self inductance matrices \([L_{ss}]\) and \([L_{rr}]\) contain only constant elements, and if \(P\) denote the number of motor poles it is clear that any inductance which is a function of angular displacement undergoes \(P/2\) complete cycles as \(\theta_{rm}\) varies from 0 to \(2\pi\). Hence the electrical displacement in electrical radians is \(\theta_e = \theta_{rm} (P/2)\). Since \(C_{em}\) is a scalar, each of the two terms comprising \(C_{em}\) must be a scalar and equal to each others. In terms of \(\theta_r\) the torque is finally given as:

\[
C_{em} = \frac{\partial W_{co}}{\partial \theta_{rm}} = \frac{P}{2} \left( \frac{\partial W_{co}}{\partial \theta_r} \right) = \frac{P}{2} \left[ I_s \right]^T \left( \frac{\partial \left[ M_{SR} (\theta_r) \right]}{\partial \theta_r} \right) \left[ I_r \right] 
\]  \hspace{1cm} (7)

3.1 Magnetically Coupled Electric Circuits State Model

Incorporating flux equation (2) in voltages equations (1) and electromagnetic torque expression (7) in the mechanical equation (4), we can write the differential equations system which governs the behavior of the dual stator induction machines.

\[
\frac{d}{dt} \left[ I \right] = \left[ L(\theta_r) \right]^{-1} \left( \left[ R \right] + \omega_r \left[ L(\theta_r) \right] \left[ I \right] + \left[ L(\theta_r) \right]^{-1} \left[ u \right] \right) 
\]  \hspace{1cm} (8)

\[
\frac{d\omega_r}{dt} = \frac{P}{2J} \left[ I \right]^T \left( \frac{d}{d\theta_r} \left[ L(\theta_r) \right] \left[ I \right] - \frac{f}{J} \omega_r - \frac{1}{J} C_{load} \right) 
\]  \hspace{1cm} (9)

At this stage a special attention is paid to the greatly dependence between the angular rotor displacement and the inductance and derivative inductance inverse matrices which their accuracy calculations decide of the accuracy and effectiveness of the magnetically electric circuits coupled model.
Equations (4)-(8) and (9) can be written as a differential algebraic state system which governs the induction machine’s:

\[
\begin{bmatrix}
  u \\
  -C_{\text{load}} \\
  0
\end{bmatrix} = \begin{bmatrix}
  \begin{bmatrix}
    L_{ss} \\
    M_{SR}(\theta_r)
  \end{bmatrix} & \begin{bmatrix}
    0 \\
    M_{SR}'(\theta_r)
  \end{bmatrix} & \begin{bmatrix}
    0 \\
    0
  \end{bmatrix}
\end{bmatrix} \begin{bmatrix}
  I^s \\
  I^r
\end{bmatrix} + \begin{bmatrix}
  0 \\
  J \\
  0
\end{bmatrix} \omega_r \begin{bmatrix}
  \theta_r
\end{bmatrix}
\]

\[
\begin{bmatrix}
  R_{ss} \\
  \omega_r \frac{d}{d\theta_r} [M_{SR}(\theta_r)]^r \\
  \frac{d}{d\theta_r} [M_{SR}(\theta_r)]^r \\
  C^s \\
  [0]
\end{bmatrix} \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  -1
\end{bmatrix} \begin{bmatrix}
  \omega_r
  \frac{d}{d\theta_r} [M_{SR}(\theta_r)]^s \\
  R_{rr} \\
  C^r \\
  f \\
  0
\end{bmatrix} \begin{bmatrix}
  I^s \\
  I^r
\end{bmatrix} + \begin{bmatrix}
  \begin{bmatrix}
    0 \\
    0
  \end{bmatrix}
\end{bmatrix} \begin{bmatrix}
  \theta_r
\end{bmatrix}
\]

To facilitate the equations treatments and their generalization for each multi-phases machines, the inductance matrices and re-written such as:

- The mutual inductance between the stator and rotor windings is expressed as:

\[
[M_{SR}(\theta)] = m_{sr} \begin{bmatrix}
  f_1^{s_1} & f_2^{s_1} & f_3^{s_1} & f_1^{s_2} & f_2^{s_2} & f_3^{s_2}
\end{bmatrix}
\]

(11.a)

where, \( f_i^{s_1} = \cos(\theta_r + \beta(i-1)) \) and \( f_i^{s_2} = \cos(\theta_r - \alpha + \beta(i-1)) \) \( i = 1, 2, 3 \)

- The derivative angular of the previous mutual inductance is expressed as:

\[
\omega_r \frac{d}{d\theta_r} [M_{SR}(\theta)] = m_{sr} \begin{bmatrix}
  g_1^{s_1} & g_2^{s_1} & g_3^{s_1} & g_1^{s_2} & g_2^{s_2} & g_3^{s_2}
\end{bmatrix}
\]

(11.b)

Where \( g_i^{s_1} = \frac{d}{d\theta_r} (f_i^{s_1}) \) and \( g_i^{s_2} = \frac{d}{d\theta_r} (f_i^{s_2}) \) \( i = 1, 2, 3 \)

And the related mechanical elements \([C^s]\) and \([C^r]\) are calculated as follows:

\[
[C^s] = \left( \frac{m_{sr}}{2} \right) [H]^{tr} \cdot [I^r]
\]

(11.c)
\[
[C^r] = \left( \frac{m_{sr}}{2} \right) [H] \cdot [I^s]
\]  

(11.d)

Where the electromechanical coupling matrix is given such as:

\[
[H] = \begin{pmatrix}
h_1^{s1} & h_3^{s1} & h_2^{s1} & h_1^{s2} & h_3^{s2} & h_2^{s2} \\
h_2^{s1} & h_1^{s1} & h_3^{s1} & h_2^{s2} & h_1^{s2} & h_3^{s2} \\
h_3^{s1} & h_2^{s1} & h_1^{s1} & h_3^{s2} & h_2^{s2} & h_1^{s2}
\end{pmatrix}
\]

(11.e)

Where \( h_i^{s1} = \sin(\theta_r + \beta(i-1)) \) and \( h_i^{s1} = \sin(\theta_r - \alpha + \beta(i-1)) \) for \( i = 1,2,3 \)

The differential equations system (10) is solved using the Rung-Kutta algorithm with variable step time correction at each rotor displacement. This permit an accuracy calculation for the model with reducing the computation cost.

4. (DSIM) Models Under Stator Inter-Turn Short Circuits And Broken Bars

4.1. (DSIM) model with broken bars

There is no need to explicitly change the topology of the equivalent circuit of the stator or rotor to take into account the phase breakdown or broken bars respectively. Inserting a higher resistance in the injured phase in the generalized model (10) during the simulation is sufficient. Generally, high power induction motors have squirrel-cage rotor, in the present work the rotor initially containing \( N_b \) bars of \( R_b \) resistance is considered as a secondary phase system composed with three windings circuits (phases). Each circuit is considered as \( N_b / 3 \) resistance connected in parallel. Thus the normal rotor phase resistance is expressed as:

\[
R_r = \frac{R_b}{\left( \frac{N}{3} \right)}
\]

(12)

When \( n_b \) defective bars induced \( (N_b / 3) - n_b \) bars in parallel, the equivalent resistance is equal to:

\[
R_r^{\text{defect}} = \frac{R_b}{\left( \frac{N_b}{3} \right) - n_b}
\]

(13)

Consequently, the increasing value of the rotor resistance is given by the difference between the health and defective resistance, explained such as:
\[ \Delta R = R_{r}^{\text{defect}} - R_r = 3R_r \frac{n_b}{N_b - 3n_b} \] (14)

The rotor matrix resistance with broken bars become:

\[
[R_{rr}]_{3 \times 3} = \begin{bmatrix}
R_{r1} + \Delta R & 0 & 0 \\
0 & R_{r2} & 0 \\
0 & 0 & R_{r3}
\end{bmatrix}
\] (15)

When one stator phase is breakdown the voltage \( u_O^S \) appear, the fault condition is take into account through increasing of the injured phase resistance until a zero current \( i_O^S \) is obtained.

### 4.2. (DSIM) under stator inter-turn short circuits

One of the most common fault in electrical motors consists of inert-turn short circuits where the motors behaves like an unbalanced load and the stator currents not constitute a balanced system. It is difficult enough to describe precisely the physical phenomenon succession that occurs in the short circuit operation. The deterioration of the insulators provide a contact between different turns for which the corresponded contact resistance becomes significantly small. The consequence of such short-circuits defect is to modify the connection between the turns of the electric circuits, and therefore to modify the number of differential equations of the new defect model. In the “Fig. 2”, the \( n_{CC} \) inter-turns coils belonging to one stator phase of the first star is represented [3],[7].

![Windings configuration of the Dual-Stator Induction Machine under inter-turn short-circuits and broken rotor windings.](image)

The added equations related to the new winding which describe the \( n_{CC} \) inter-turn short
circuits fed by \( u_{cc}^s \) voltage and \( i_{cc}^s \) current of short-circuits coils of one stator phase containing \( n_s \) turn are:

\[
u_{cc}^s = R_{cc} \cdot I_{cc} + \frac{d}{dt} [\phi_{cc}]
\]  \( \text{(16)} \)

\[
\begin{aligned}
[\phi] &= \begin{bmatrix}
[L_{ss}]_{6\times6} & [M_{SR}(\theta_r)]_{3\times6} & \left(M(\theta_{cc})\right)_{cc}^{s} & \left(M(\theta_{cc})\right)_{cc}^{r} & \left(I_{cc}^s\right) \\
[M_{SR}(\theta_r)]_{3\times6} & [L_{rr}]_{3\times3} & \left(M(\theta_{cc})\right)_{cc}^{s} & \left(M(\theta_{cc})\right)_{cc}^{r} & \left(I_{cc}^r\right) \\
\end{bmatrix} \\
\end{aligned}
\]  \( \text{(17)} \)

\[
\begin{aligned}
[\phi_{cc}] &= \begin{bmatrix}
\left(M(\theta_{cc})\right)_{cc}^{s} & \left(M(\theta_{cc})\right)_{cc}^{r} & \left(L_{cc}\right)_{1\times1} & \left(I_{cc}^s\right) \\
\end{bmatrix} \\
\end{aligned}
\]  \( \text{(18)} \)

The additional elements of the resistances and inductances matrices due to the short circuits percent \( \eta_{cc} \) are given as follows:

\[
\begin{aligned}
[R_{cc}] &= \eta_{cc} \cdot R_{a1}^s \\
[L_{cc}]_{1\times1} &= (\eta_{cc})^2 \cdot (L_{a1}^s) \\
\end{aligned}
\]  \( \text{(19.a)} \)

\[
\begin{aligned}
\left[M(\theta_{cc})\right]_{cc}^{s} &= \eta_{cc} \cdot (m_{s1s2}) \\
\left[M(\theta_{cc})\right]_{cc}^{r} &= \eta_{cc} \cdot (m_{sr}) \\
\left[M(\theta_{cc})\right]_{cc}^{s/r} &= \begin{bmatrix}
\left(M(\theta_{cc})\right)_{cc}^{s} \\
\left(M(\theta_{cc})\right)_{cc}^{r}
\end{bmatrix}
\end{aligned}
\]  \( \text{(19.c)} \)

\[
\begin{aligned}
[\left[M(\theta_{cc})\right]_{cc}^{s/r} &= \begin{bmatrix}
\cos(\theta_{cc}) \\
\cos(\theta_{cc} + \beta) \\
\cos(\theta_{cc} - \beta) \\
\cos(\theta_{cc} - \alpha) \\
\cos(\theta_{cc} - \alpha + \beta) \\
\cos(\theta_{cc} - \alpha - \beta)
\end{bmatrix} \\
\left[M(\theta_{cc})\right]_{cc}^{s/r} &= \begin{bmatrix}
\cos(\theta_{cc} - \theta_{r}) \\
\cos(\theta_{cc} - \theta_{r} + \beta) \\
\cos(\theta_{cc} - \theta_{r} - \beta)
\end{bmatrix} \\
\end{aligned}
\]  \( \text{(19.d)} \)

\[
\begin{aligned}
\left[M(\theta_{cc})\right]_{cc}^{s/r} &= \begin{bmatrix}
\left[M(\theta_{cc})\right]_{cc}^{s} \\
\left[M(\theta_{cc})\right]_{cc}^{r}
\end{bmatrix}
\end{aligned}
\]  \( \text{(19.e)} \)
Where \( \eta_{cc} = \frac{n_{cc}}{n_s} \) is percent of inter-turn short circuit and the injured phase is associated with the \( \theta_{cc} \) angle [7].

Incorporating flux equations (17), (18) in the voltages equations (1), (16), and electromagnetic torque expression (7) including implicitly the short circuit effect through the currents in the mechanical equation (4), we can write the differential equations system which governs the behaviour of the dual stator induction machines.

\[
\frac{d}{dt} \begin{bmatrix} I_s^r \\ I_s^s \end{bmatrix} = \begin{bmatrix} [L(\theta_r)] & [M(\theta_r)]_{rr}^s \\ [M(\theta_r)]_{rr}^s & [L_{cc}] \end{bmatrix}^{-1} \times \begin{bmatrix} u_s^r \\ u_s^s \end{bmatrix} + \begin{bmatrix} \omega_r \frac{d}{d\theta_r} [L(\theta_r)] & [M(\theta_r)]_{rr}^s \\ [M(\theta_r)]_{rr}^s & [L_{cc}] \end{bmatrix} \begin{bmatrix} I_s^r \\ I_s^s \end{bmatrix} \]

\[
\frac{d\omega_r}{dt} = \frac{P}{2J} \begin{bmatrix} I_s^r \end{bmatrix} \begin{bmatrix} [L(\theta_r)] & [M(\theta_r)]_{rr}^s \\ [M(\theta_r)]_{rr}^s & [L_{cc}] \end{bmatrix} \begin{bmatrix} I_s^r \\ I_s^s \end{bmatrix} - \frac{f}{J} \omega_r - \frac{1}{J} C_{load}
\]

Equations (20), (21) and (9) can be written as a differential algebraic state system which governs the dual stator induction machine’s under one phase stator inter-turn short circuits:

\[
\begin{bmatrix} u_s^r \\ u_s^s \\ u_r^s \\ u_r^r \\ -C_{load} \\ 0 \end{bmatrix} = \begin{bmatrix} [L_{ss}] & [M_{SR}(\theta_r)] & [M(\theta_r)]_{rr}^s \\ [M_{SR}(\theta_r)] & [L_{rr}] & [M(\theta_r)]_{rr}^r \\ [M(\theta_r)]_{rr}^s & [L_{cc}] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [0] \end{bmatrix} \begin{bmatrix} I_s^r \\ I_s^s \\ I_r^r \\ I_r^s \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_r \frac{d}{d\theta_r} [L_{cc}] & [0] \\ [0] & \omega_r \frac{d}{d\theta_r} [M_{SR}(\theta_r)]_r \\ [0] & \omega_r \frac{d}{d\theta_r} [M(\theta_r)]_{rr}^r \\ [0] & \omega_r \frac{d}{d\theta_r} [M(\theta_r)]_{rr}^s \\ 0 & \omega_r \frac{d}{d\theta_r} [C_{rr}] \\ 0 & \omega_r \frac{d}{d\theta_r} [C_{rr}] \end{bmatrix} \begin{bmatrix} I_s^r \\ I_s^s \\ I_r^r \\ I_r^s \\ 0 \end{bmatrix} \]

\[
\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} \omega_r \frac{d}{d\theta_r} [L_{cc}] & [0] \\ [0] & \omega_r \frac{d}{d\theta_r} [M_{SR}(\theta_r)]_r \\ [0] & \omega_r \frac{d}{d\theta_r} [M(\theta_r)]_{rr}^r \\ [0] & \omega_r \frac{d}{d\theta_r} [M(\theta_r)]_{rr}^s \\ 0 & \omega_r \frac{d}{d\theta_r} [C_{rr}] \\ 0 & \omega_r \frac{d}{d\theta_r} [C_{rr}] \end{bmatrix} \begin{bmatrix} I_s^r \\ I_s^s \\ I_r^r \\ I_r^s \\ 0 \end{bmatrix} \]

(22)
The combination of the equations (1)-(2), and (3)-(5) with the short circuits equations (11)-(17), leads to the final reduced multi-turn state formulation model (19) of the (DSIM) under short-circuit turns of one stator star. The differential equations system (20) is solved using the fourth order Runge-Kutta method while the step time is chosen at each time as the inverse of the maximal eigen values of the state matrix.

5. Simulations Results and Discussions

To simulate the (DSIM) model, a native open computer program implemented under Matlab is used to perform calculation of inductances matrices and to simulate transient and steady state behavior of the (DSIM) were the main state formulation of the differential equations systems (10 ) and (19) are solved using the fourth order Runge-Kutta method.

The induction motor parameters are detailed in the appendix. The simulation results of the transient and steady state behaviour of the motor are obtained after numerically solving the differential algebraic equations system (10). The model program implemented under Matlab environment is open for interfaced with commercial softwares. The stator windings studied defects are different number of turn-to-turn short circuits. The analysed quantities are the speed, torque and the stator/rotor currents.

5.1. Stator Inter-turn shorts circuits faults

Consider a case of inter-turn short circuits of one star of the stator phases. It mean that 5% turns of the phase “a” are shorted. The motor operate at its nominal state with a load torque imposed at \( t = 0.8 \) s. While the loaded motor steady state operation is achieved, a short-circuit of 5% turns is imposed at \( t = 1.25 \) s.

The obtained results for simulated operating modes, healthy no-load, healthy load and inter-turns short circuits with loaded (DSIM) are given by the following "Figs. 3-12". The results concern the studied quantities: speed, torque, stator and rotor phase currents.

![Fig. 3: Speed under healthy and 5% inter-turns short-circuits of the loaded (DSIM).](image-url)
Fig. 4: Torque under healthy and 5% inter-turns short-circuits of the loaded (DSIM).

Fig. 5: Current of the first star phase Sa1 current under healthy and 5% inter-turns short-circuits.
Fig. 6: Current of first star phase Sb1 under healthy and 5% inter-turns short-circuits.

Fig. 7: Current of first star phase Sc1 under healthy and 5% inter-turns short-circuits.
Fig. 8: Current of second star phase Sa2 under healthy and 5% inter-turns short-circuits.

Fig. 9: Current of second star phase Sb2 under healthy and 5% inter-turns short-circuits.
Fig. 10: Current of second star phase Sc2 under healthy and 5% inter-turns short-circuits.

Fig. 11: Current of rotor phase Ra under healthy and 5% inter-turns short-circuits.
Fig. 12: Current of rotor phase Rb under healthy and 5% inter-turns short-circuits.

Fig. 13: Current of rotor phase Rc under healthy and 5% inter-turns short-circuits.

Figs. 3, and 14, and Fig.4, show that the magnitude of the speed and torque signals increased at every inter-turn short circuits. From the Fig.5 to Fig.10 of the stator currents, we can see that the current of the phases ‘a’ and ‘b’ related to the two stars increased, the opposite effect is observed on the current of the phases ‘c’ where their decreasing behavior can cause a great unbalance on the currents system and as we have above mentioned a torque and speed oscillations become more important.

Fig. 11, 12 and 13 show the rotor phases currents from which we observe important increasing values combined to significant modulation.

The steady state speed and stator currents results for different inter-turns short-circuits percent are summarized in the "Fig. 14," and "Fig. 15."
Fig. 14: Steady state speed of the loaded (DSIM) under different percent of inter-turns short-circuits.

Fig. 15: Steady state stator phases current of the loaded (DSIM) under different percent of inter-turns short-circuits.

From the steady state currents given by the Fig.15 for different inter-turns short circuits percent, the tendency is an increase of the currents, this indicates that the current flowing into the short circuit rises to an extremely high value, and multiplying the risk of further degradation or even a motor deterioration.

5.2. Stator phase breakdown fault

Consider a case of one stator broken phase inter-turn short circuits of one star of the stator phases. It means that the resistance of the injured first star phase “a” is increased (average $20 \cdot R_{a1}^g$) until obtained an zero current. The motor operate at its nominal state with a load torque imposed at $t = 0.75s$, and the related fault imposed at $t = 1.25s$. The obtained results with loaded (DSIM) concerning the torque, stator and rotor phase currents are given by the following "Figs. 19-20".
Fig. 16: Electromagnetic torque under stator phase breakdown.

Fig. 17: First star of stator currents under stator phase breakdown.

Fig. 18: Second star of stator currents under stator phase breakdown.
Fig. 19: Rotor currents under stator phase breakdown.

Fig. 16 show that the magnitude of the torque increase slightly but important oscillation appears. From the Fig.17 and Fig.18 of the stator currents, we can see that the current of the phases ‘b1,c1,a2,b2’ related to the two stars increase significantly, but the opposite effect is observed on the current of the phases ‘c2’ where the current is unchanged.

5.3. Rotor broken bars

The last studied fault concern the broken of two rotor bars corresponding to increase the rotor phase resistance about $\Delta R = 0.9\Omega$. The motor operate at same previous conditions but with the fault imposed at $t = 1.25s$. The obtained results with loaded (DSIM) are given by the following "Figs. 20-24". The results concern the studied quantities: speed, torque, stator and rotor phase currents.

Fig. 20: (DSIM) speed at healthy and broken rotor bars.
Fig. 21: (DSIM) electromagnetic torque at healthy and broken rotor bars.

Fig. 22: (DSIM) currents first stator star phases at healthy and broken rotor bars.

Fig. 23: (DSIM) currents of the second stator star phases at healthy and broken rotor bars.
Fig. 24: (DSIM) currents of the rotor phases at healthy and broken rotor bars.

Fig. 20 and Fig. 21, show the apparition of oscillations of the speed and torque. From the Fig. 22 and Fig. 23 of the stator currents, we can see that although the currents magnitude increase slightly, the signals are greatly modulated. In the Fig. 24, the rotor current corresponded to the injured phases decrease, and any significant changes observed on the other currents.

6. Conclusion

An accurate transient coupled magnetic electric circuit model of Dual-Stator-Induction-Machines under healthy and stator inter-turns short circuits, stator breakdown and broken rotor bars faults has been presented in this paper. Based on the actual windings configuration without any transformation or equivalent variables, the state formulation model is obtained from the electric circuits equations magnetically coupled through the flux equations. Both equations system associated with the mechanical ones through the torque computation leads to two-axis state generalized models of (DSIM) under healthy and faults operating conditions. The developed models implemented under Matlab environment are used to simulate the motor behavior under different ratio turn-to-turn short circuits of the stator phase, and the broken rotor bars or stator windings breakdown are also studied. The investigated quantities are the stator and rotor currents, electromagnetic torque and mechanical speed. It is worth noticing that with the proposed robust model many other faults such as two simultaneous stator inter-turns short circuits, stator and current breakdown, can be studied because of its accuracy and its versatility.

Appendix

The (DSIM) parameters are:

\[
\begin{align*}
V &= 220/380 V, \\
A &= 5/8.5 A, \\
R_s &= 1.86 \Omega, \\
R_r &= 2.12 \Omega, \\
L_s &= 110.5 mH, \\
L_r &= 1.7 mH, \\
L_{fs} &= 0.8 mH, \\
L_{fr} &= 60 mH, \\
p &= 2, \\
M_{sr} &= 72.6 mH, \\
J &= 0.0625 Kg \cdot m^2, \\
C_{load} &= 15 Nm, \\
f &= 0.0086 Kg m^2 \cdot s^{-1}, \\
Frequency &= 50 Hz.
\end{align*}
\]
References


