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Regular paper

Blind signal separation of harmonic voltages in non-linear loads

Extensive use of non linear loads like arc furnace and PWM Inverters generate considerable voltage harmonics. These harmonics need to be estimated and quantified using minimum knowledge about the topology of the system. Blind Signal Processing Techniques like Fast ICA (FICA), Joint Approximate Diagonalisation of Eigen Matrices (JADE) and Entropy Bound Minimisation (EBM) are applied for harmonic voltage estimation in a simple four bus system. Conventional ICA algorithms like FICA and EBM algorithm break down when only the arc furnace load contributes to the harmonics in the power system. An algorithm termed as FICOMB which is a combination of Fast ICA and COMBI is employed for harmonic voltage estimation. The graphical results and error indices of a simple four bus system indicate that FICOMB and JADE are suitable for harmonic frequencies like h = 3 and EBM is suitable for h = 5. The primary objective of the present work is to use the estimation further for either the design of mitigation equipment or for identifying the perpetrators of harmonics with accuracy being more significant for both the applications. This calls for an optimal estimator to choose a definite algorithm for a particular frequency.

Keywords: FICOMB; PWM Inverter; Arc furnace; AAPE Error.

1. Introduction

The advent of power electronics has resulted in the generation and propagation of harmonics in power systems. The harmonics are in the form of harmonic currents as well as harmonic voltages. Estimation of harmonic voltage components in the frequency domain uses methods like the DFT algorithm, Goertzel algorithm and parameter estimation methods like ESPRIT, MUSIC etc. and combined three–four parameter sine fitting algorithm, non-iterative multi harmonic fitting algorithm in the time domain[1]. When the voltage estimation of various harmonic components is to be carried out in a state estimation environment for interconnected systems, ICA is found to be highly suitable. A few non linear loads like arc furnace, PWM Inverters, etc are dominant sources of voltage harmonics. The estimation of such harmonics is generally cumbersome owing to the characteristics of arc furnaces which generate substantial even order harmonics and certain odd order harmonics like h = 3, 9,etc which are not conventionally present in other non linear loads. Blind Signal Processing has been used for harmonic current estimation [2-4]. It is extended to harmonic voltage estimation using two Independent Component analysis (ICA) algorithms namely Fast ICA (FICA) [5,6] and Joint Approximate Diagonalisation of Eigen Matrices (JADE) [7] in [8]. When harmonics exist at one bus only due to the arc furnace for specific harmonics, and the remaining buses do not contribute to the harmonics, it is observed that FICA breaks down. However, JADE is found to be precise for such a case. In this work, the effect of voltage harmonics employing a combination of FICA and COMBI [9 - 11] called as FICOMB is found to have an equivalent performance as JADE for such a work. One of the most accurate ICA algorithms called as Entropy Bound Minimisation (EBM) [12, 13] is also analysed in the work. But a deeper analysis indicates that this algorithm is not appropriate for voltage harmonic estimation of non linear loads like arc furnaces, which have considerable harmonic content at certain non–existent
frequencies like \( h = 2, 3, 4, \) etc. It is also observed that FICOMB has better accuracy at all harmonic frequencies and also its separation quality is better.

2. Notation

The notation used throughout the paper is stated below.

**Indexes:**
- \( b \) bus number
- \( h \) harmonic frequency
- 1 source signal 1
- 2 source signal 2

**Constants:**
- \( n \) number of buses
- \( x \) observation vector [volts]
- \( A \) mixing matrix
- \( s \) source vector
- \( V_{bh} \) harmonic voltage injected at the various buses ‘\( b \)’ at various ‘\( h \)’ [volts]
- \( Z_h \) impedance at the harmonic frequency ‘\( h \)’
- \( I_{bh} \) harmonic current at the various buses ‘\( b \)’ at various ‘\( h \)’ [Amperes]
- \( Y \) output vector
- \( W \) demixing matrix
- \( ISR \) interference to signal ratio
- \( I \) mutual information
- \( H \) entropy
- \( e_h \) error between actual and estimated voltage
- \( MAE \) mean absolute error
- \( AME \) average mean error
- \( MSE \) mean square error
- \( AAPE \) average absolute percentage error

3. Blind Source Separation

3.1. Problem Definition

Blind Signal Separation (BSS) is a statistical decomposition technique in which the independent signals are extracted from the observed signals using statistical independence as the main criteria. The ICA estimates the sources \( s \ (n) = [s_1 (n) \ s_2 (n) \ldots \ s_n (n)] \) from \( n \) observed signals, \( x \ (n) = [x_1(n) \ x_2(n) \ldots \ x_n (n)] \). The noiseless linear model of ICA problem is assumed as

\[
x = As
\]

(1)

where \( A \) is an \( n \times n \) mixing matrix, \( s \) is an original source vector which has \( n \) independent components and \( x \) is an \( n \times 1 \) observation vector. The block diagram representation of BSS is shown in fig. 1.
The objective of BSS is to determine an optimised value of \( A \). This principle can be applied to harmonic voltage estimation as given below.

For an \( n \)-bus system, the Harmonic State Estimation (HSE) problem is defined as:

\[
V_{bh} = Z_h I_{bh}
\]  

(2)

where \( V_{bh} \) is the harmonic voltage injected at the various buses ‘\( b \)’ (vector of \( n \times 1 \)) at harmonic frequency ‘\( h \)’, \( I_{bh} \) is the harmonic current at the various buses ‘\( b \)’ (vector of \( n \times 1 \)) at harmonic frequency ‘\( h \)’ and \( Z_h \) is the impedance of the system at harmonic frequency ‘\( h \)’. This model has been formulated without considering the effect of noise on the power system.

The model given in (2) is significant since two unknowns namely \( A \) and \( s \) are to be determined from one known value. A solution to such a problem is obtained using some information on the statistical properties of the signal’s (i.e., the signals are statistically independent) and by optimising certain contrast function or objective function called as the score function.

The output vector is represented as \( Y \). That is, the output will be given by

\[
Y = Wx
\]  

(3)

where \( Y \) is an estimate of the sources and \( W \) is a demixing matrix with the same dimensions as \( A \).

From (2) and (3), it is clear that the condition \( s = Y \) occurs, only when \( A = W^{-1} \). The BSS algorithm aims at finding a matrix \( W \) to undo the mixing effect and thereby to satisfy the condition \( x = Y \).

Usually the information on the topology of the power system network may be available but it is difficult to obtain the \( Y_h \) values at different frequencies. The usage of BSS to HSE is thus justified as BSS computes each \( V_{bh} \) with sufficient accuracy employing the available \( I_{bh} \) values.

3.3. Blind Signal Processing Algorithms
The three ICA algorithms used for the work are discussed for completeness.
3.3.1 Fast ICA: This algorithm is explained in detail in [2-4].
3.3.2 COMBI: COMBI is an algorithm which combines both EFICA and WASOBI (weight adjusted variant of SOBI) given in [8]. EFICA [9-10] is a higher order statistics based algorithm and WASOBI is a time structured second order based algorithm. It is an improvement over the second order blind identification (SOBI) in the form of weight
adjustment. The main technique is based on extracting the non–gaussian sources using EFICA and the extraction of the remaining sources using WASOBI. The algorithm is implemented as follows:

1. Apply both EFICA and WASOBI to the observation vector $x$. Obtain the extracted source signals as $s_1^1$ and $s_2^2$ respectively. Evaluate the ISR matrix $ISR_1^1$ and $ISR_2^2$ and the corresponding vectors $isr_1^1$ and $isr_2^2$.

2. Calculate the minimum $isr_1^1$ and $isr_2^2$ and represent it as $A = isr_1^1$ and $B = isr_2^2$.

3. Based on the values of $A$ and $B$, signals are chosen and discarded. When $isr_1^1 < A$ for $s_1^1$ accept the signals and redefine $z$ as the discarded signals of $s_1^1$. Similarly evaluate the chosen signals for $s_2^2$ (i.e.) $isr_2^2 < B$ and redefine $z$ as the discarded signal of $s_2^2$.

4. If from (3), there is more than one rejected signal repeat from step 1. Else accept the rejected signal.

3.3.3 FICOMB: An integration of FICA and COMBI results in FICOMB. The observed signals are processed by FICA and then by COMBI. Such a signal is able to extract the source signal at all harmonic frequencies.

3.3.4 JADE: JADE ICA is rooted in the joint diagonalization of cumulant matrices. This algorithm utilises the second and fourth order cumulants for source separation. More details of the algorithm are found in [6, 7]. JADE ICA is attempted as it is found to be very accurate and its separation level is very high under almost all harmonic frequencies, even with one source alone being active.

3.3.4 EBM: The ICA-EBM algorithm uses a line search procedure which utilises the updates that constrain the demixing matrix to be orthogonal. For ICA, in order to separate $n$ signals the mutual information $I(y_1; y_2; \ldots; y_n)$ among $n$ random variables $y_n$, $n=1, 2, \ldots, n$ is given in (4).

$$I(y_1; y_2; \ldots; y_n) = \sum_{i=1}^{N} H(y_i) - \log |\det(W)| - H(x)$$

where $H(y_n)$ is the entropy of the $n^{th}$ separated source and $H(x)$ the entropy of observations which is a constant with respect to $x$. More details of the algorithm are given in [8] and [9]. Thus, this entropy uses the tightest maximum entropy bound for implementing ICA algorithm.

4. Methodology of Implementation:

A four bus system used as the test sample for the work is shown in fig. 2 which is a modified bus system given in [13].

![Four Bus Test System](image-url)
The four bus test system comprises two generators, two linear loads, two non-linear loads generating considerable voltage harmonics and four transmission lines linking the four buses. The harmonic voltage injected into the system is represented by two harmonic voltage sources, PWM Inverter at bus two and an arc furnace at bus three. The transmission line parameters used for simulation are specified in Table 1. The two generators used in the model have a p.u. value of 1.

### Table 1: Transmission parameters at harmonic frequencies [p.u.]

<table>
<thead>
<tr>
<th>S.No</th>
<th>Line p-q</th>
<th>Series Impedance</th>
<th>Shunt admittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-4</td>
<td>0.02+jh 0.06</td>
<td>jh0.03</td>
</tr>
<tr>
<td>2</td>
<td>1-2</td>
<td>0.08+jh0.24</td>
<td>jh0.025</td>
</tr>
<tr>
<td>3</td>
<td>2-3</td>
<td>0.08+jh0.24</td>
<td>jh0.025</td>
</tr>
<tr>
<td>4</td>
<td>3-4</td>
<td>0.04+jh0.012</td>
<td>jh0.015</td>
</tr>
</tbody>
</table>

The operating cycle of an arc furnace consists of a melting stage and numerous refining stages. The abnormal behaviour of the arc furnace is due to the random movement of the melting material which causes the even harmonics also to exist. The refining stage is accompanied by a large arcing operation. Thus, the arc furnace becomes a highly varying load which pollutes the entire system. Though, several papers on arc furnace modelling are available, a static harmonic model used during the melting and refining stage is given in [16]. This information is utilized to create a random profile for the a.c operated arc furnace. The second, third, and fifth harmonics are dominant during both the stages.

In a PWM inverter [15], the inverter output has to be sinusoidal with both magnitude and frequency being controllable. To obtain a sinusoidal output voltage at a desired frequency, a sinusoidal control signal at the desired frequency is compared with a triangular waveform. The switching frequency of the inverter is based on the frequency of the triangular waveform and is normally kept constant along with its amplitude $v_{tri}$. The triangular waveform $v_{tri}$ operates at a frequency $f_s$ which decide the frequency with which the inverter switches are switched. The control signal $v_c$ is used to modulate the switch duty ratio and has a frequency $f_1$ which is the desired fundamental frequency of the inverter output. The amplitude modulation ratio is given by $m_a = v_c/v_{tri}$. The frequency modulation ratio is defined as $m_f = f_s/f_1$. An over modulation ($m_a>1$) scheme is used in PWM, so that the amplitude of the fundamental frequency component in the output voltage is increased to a value greater than 1. This causes more harmonics to appear in the voltage. It is this case which is implemented in the work in [8].

Initially, in order to perform the HSE, prior information about the location of the harmonics is assumed apriority or based on the total voltage distortion. As a first step, a fundamental load flow analysis is conducted using Newton Raphson Method. This gives the fundamental voltage values at the various buses. Using these fundamental voltage values, modelling of the linear loads and non-linear loads the methodology is carried out as elaborated in [4].
The harmonic analysis is implemented using the relation given in (3). The equation (3) bears closeness to equation (1) which is the basic ICA model expression. With the ICA model, the value of $V_{bh}$ can be evaluated from the measured sensor readings ($I_{bh}$).

To perform the simulation of the four bus model the observation data needs to be produced and a generalised method of implementation needs to be followed. The observation data (harmonic current) is generated in the following manner:

1. The linear loads and the other power system components of the four bus system is modelled as constant impedance loads as in [14]. The harmonic source is modelled as a harmonic voltage source in [4]. Thus the admittance of the test system is formulated for a specific harmonic frequency say ($h = 2$).

2. The harmonic voltage of the arc furnace at bus 3 is computed at $h = 2$ frequency while the harmonic voltage values at the remaining buses are taken as zero and a single harmonic current data is generated for the system at $h = 2$.

3. A random profile for the various buses is modelled using the Markov model based on Poisson distribution concept for the four bus system. Using this profile, the harmonic voltage data is generated at buses two and three for 1440 readings. Substituting these readings for $V_{bh}$ and $Y_h$ in the dual of equation (2) defined as: $I_{bh} = Y_h V_{bh}$, 1440 values for $I_{bh}$ are obtained at $h = 2$.

4. Steps (1), (2) and (3) are repeated for $h = 3, 4$. The corresponding $I_{bh}$ data generated are defined as the harmonic current sensor readings. The arc furnace generates appreciable harmonics at $h = 2, 3$ and $h = 4$. With the harmonic voltage value for the arc furnace at Bus 3 and the remaining harmonic voltages as zero, the harmonic current data is generated at $h = 2$ for the four bus system. Using step (3) 1440 values of harmonic current data for the arc furnace is generated at various frequencies. For the non-triplen harmonic frequencies, the PWM inverter at Bus 2 is also modelled as in Step 3 and Step 4.

The observed data for the four bus system is obtained at each harmonic frequency by back substitution method as no specific data are available for the voltage readings. A random current profile with 1440 readings is thus generated using the two harmonic voltage sources namely the PWM Inverter and Arc Furnace at buses two and three respectively. Taking into account, one reading in one minute for one day the value of 1440 readings is obtained. By employing the voltage profile and the impedance of the system, the current readings are computed which serve as the observation data. The methodology of implementation is given as follows:

- **Step 1:** Determine the Harmonic Injection Buses (HIB) using total voltage harmonic distortion
- **Step 2:** Obtain the sensor readings at the buses assuming the two harmonic voltage sources on a specific day.
- **Step 3:** Apply FICOMB algorithm.
- **Step 4:** Compute MAE, AME, MSE and AAPE.
- **Step 5:** Repeat step (2) and step (3) for JADE and EBM algorithms.
- **Step 6:** Repeat for all harmonics.
5. Simulation Results
The sensor readings are recorded and analysed on a specific day exhaustively for the harmonic frequencies \( h = 2, 3, 4, 5, 7, 11, 13 \) and 17 respectively and the voltages at buses two and three are estimated for each of these harmonic frequencies. The estimated voltages \( V_{eh} \) for that particular day are portrayed in figure three at buses two and three for \( h = 3 \) only as this harmonic frequency is a non-conventional one. The real and imaginary components of harmonic voltages for the three algorithms-FICOMB, EBM and JADE are thus plotted.

![Figure 3: Harmonic Load Voltages at Buses 2 and 3 for \( h = 3 \)](image)

From the graphs in Fig. 3 it is observed that for \( h = 3 \), the FICOMB and JADE have similar performance and separate the estimated voltage accurately. However, EBM does not provide reliable results for \( h = 3 \). The variation between the estimated and actual parameters is computed using four error indices defined as mean absolute error (MAE), average mean error (AME), mean square error (MSE) and average absolute percentage error (AAPE).

These quantities are defined as:

\[
\text{MAE} = \max(e_h(t))_{t=1,2,\ldots,T} \tag{5}
\]

\[
\text{AME} = \frac{1}{T} \sum_{t=1}^{T} |e_h(t)| \tag{6}
\]

\[
\text{MSE} = \frac{1}{T} \sum_{t=1}^{T} |e_h(t)|^2 \tag{7}
\]

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AAPE is defined in terms of the error $e_h$ which is the error between the actual harmonic voltage $V_{ah}$ and the estimated harmonic voltage $V_{eh}$ ($e_h = V_{ah} - V_{eh}$).

$$AAPE(\%) = \frac{1}{T} \left| \frac{e_h(t)}{V_{ah}(t)} \right| \times 100\%$$

The value of AAPE serves as the separation index of the algorithm. These error indices are computed for $h = 3$ and for $h = 5$ at buses two and three respectively and are shown in Table 2 and Table 3 respectively.

**Table 2: Error at $h = 3$ and $h = 5$ for PWM Inverter**

<table>
<thead>
<tr>
<th>Error</th>
<th>$h = 3$(real)</th>
<th>$h = 5$(real)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>FICOMB</td>
<td>EBM</td>
</tr>
<tr>
<td></td>
<td>1.00E-05</td>
<td>1.00E-05</td>
</tr>
<tr>
<td>AME</td>
<td>3.00E-06</td>
<td>3.00E-06</td>
</tr>
<tr>
<td>MSE</td>
<td>2.00E-11</td>
<td>2.00E-11</td>
</tr>
<tr>
<td>AAPE</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 3: Error at $h = 3$ and $h = 5$ for Arc Furnace**

<table>
<thead>
<tr>
<th>Error</th>
<th>$h = 3$(real)</th>
<th>$h = 5$(real)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>FICOMB</td>
<td>EBM</td>
</tr>
<tr>
<td></td>
<td>4.00E-05</td>
<td>2.00E-05</td>
</tr>
<tr>
<td>AME</td>
<td>1.00E-05</td>
<td>8.00E-06</td>
</tr>
<tr>
<td>MSE</td>
<td>3.00E-10</td>
<td>9.00E-11</td>
</tr>
<tr>
<td>AAPE</td>
<td>0.1504</td>
<td>66.62</td>
</tr>
</tbody>
</table>

Using AAPE error as the real error index, it is found that for $h = 3$, FICOMB and JADE have an equivalent performance. An in depth analysis of the tables 2 and 3, implies that EBM has the lowest AAPE value at $h = 5$ which calls for a detailed investigation in choosing the best ICA algorithm at different frequencies.
6. Conclusions
A blind signal processing based algorithm for specific loads like arc furnace, PWM Inverters, etc in a simple four bus system is attempted in this work. A combined algorithm of FICA and COMBI termed as FICOMB is developed which has an equivalent performance as JADE, especially for h = 3 when the arc furnace is the only load present in the power system and other conventional ICA algorithms are unsuccessful. At normal conditions when minimum of two non linear loads exist the accuracy of the EBM algorithm is distinguishable from the AAPE value for this algorithm. Hence, an optimal estimator is crucial for harmonic voltage estimation based on blind signal processing algorithms if such a scheme is to be integrated to a smart grid environment for analysing power quality.

References